Fermions are not Qubits

musings about fermionic mode entanglement

Seminar Talk, Université de Genève

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Motivation

- Fermions fundamental part of description of nature
  - essentially all (standard) matter consists of fermions
- Effects of QFT on curved spacetimes, e.g., Unruh/Hawking
  - create mode entanglement
- Progress in simulations and electron quantum optics

Want to use fermionic modes in quantum information, but how?

"Fermions are like qubits . . . aren't they?"

- Pauli principle ⇒ two-level systems
  - fermionic mode k either occupied, $|1_k\rangle$, or not, $|0_k\rangle$
- For fixed number of modes ∃ isomorphism:
  - Fock space ⇒ tensor product space
- QI: also need consistent notion of subsystems ⇒ Trouble[1]

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- Fermionic Gaussian states:

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Eisler & Zimborás, *On the partial transpose of fermionic Gaussian states*, New J. Phys. **17**, 053048 (2015)

- Teleportation, Computation, etc.

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- Description of fermions: states on Fock spaces
- Fermionic ambiguity—issues with partial trace
- Consistency conditions for subsystems
- Consistent mappings between fermions and qubits?
- Role and origin of superselection rules
- Implications for fermions in quantum information

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Fermions in a nutshell

Fermionic Fock space 101

Single-fermion wave functions ψ_m - modes - associate $a_m, a_m^\dagger$

anticommutation relations $\{a_m, a_n^\dagger\} = \delta_{mn}, \ \{a_m, a_n\} = 0$

vacuum state $|\,0\,\rangle \in \mathcal{H}_{\text{0-p}}$, annihilated by all a_m, $a_m\,|\,0\,\rangle = 0 \ \ \forall\, m$

single-particle states: $|\,1_m\,\rangle = |\,\psi_m\,\rangle = a_m^\dagger\,|\,0\,\rangle \in \mathcal{H}_{\text{1-p}}$

two-particle states $\in \bar{S}(\mathcal{H}_{\text{1-p}} \otimes \mathcal{H}_{\text{1-p}})$

$$|\,1_m, 1_n\,\rangle = a_m^\dagger a_n^\dagger\,|\,0\,\rangle = \frac{1}{\sqrt{2}}\Big(|\,\psi_m\,\rangle \otimes |\,\psi_n\,\rangle - |\,\psi_n\,\rangle \otimes |\,\psi_m\,\rangle\Big)$$

N-mode Fock space $\mathcal{F}_N(\mathcal{H}_{\text{1-p}})$

$$\mathcal{F}_N(\mathcal{H}_{\text{1-p}}) = \bigoplus_{m=0}^{N} \bar{S}\Big(\mathcal{H}_{\text{1-p}}^{\otimes m}\Big) = \mathcal{H}_{\text{0-p}} \oplus \mathcal{H}_{\text{1-p}} \oplus \bar{S}\Big(\mathcal{H}_{\text{1-p}} \otimes \mathcal{H}_{\text{1-p}}\Big) \oplus \ldots$$

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$$\mathcal{F}_N(\mathcal{H}_{\text{1-p}}) = \bigoplus_{m=0}^{N} \bar{S}\Big(\mathcal{H}_{\text{1-p}}^{\otimes m}\Big) = \mathcal{H}_{\text{0-p}} \oplus \mathcal{H}_{\text{1-p}} \oplus \bar{S}\Big(\mathcal{H}_{\text{1-p}} \otimes \mathcal{H}_{\text{1-p}}\Big) \oplus \dots$$

Density operators on the Fock space

General pure state on the Fock space

$$|\,\psi\,\rangle = \mu_0\,|\,0\,\rangle + \sum_{i\neq 0}\mu_i\,|\,1_i\,\rangle + \sum_{i,j}\mu_{ij}\,|\,1_i,1_j\,\rangle + \dots$$

Mixed states $\Rightarrow$ convex sums of projectors

Example: two-mode mixed state

$$\begin{aligned}\rho_{kk'} = {} & \alpha_1\,|\,0\,\rangle\langle\,0\,| + \alpha_2\,a_{k'}^\dagger\,|\,0\,\rangle\langle\,0\,|\,a_{k'} + \alpha_3\,a_k^\dagger\,|\,0\,\rangle\langle\,0\,|\,a_k \\ & + \alpha_4\,a_k^\dagger a_{k'}^\dagger\,|\,0\,\rangle\langle\,0\,|\,a_{k'}a_k + \big(\beta_1\,|\,0\,\rangle\langle\,0\,|\,a_{k'} + \beta_2\,|\,0\,\rangle\langle\,0\,|\,a_k \\ & + \beta_3\,|\,0\,\rangle\langle\,0\,|\,a_{k'}a_k + \beta_4\,a_{k'}^\dagger\,|\,0\,\rangle\langle\,0\,|\,a_k + \beta_5\,a_{k'}^\dagger\,|\,0\,\rangle\langle\,0\,|\,a_{k'}a_k \\ & + \beta_6\,a_k^\dagger\,|\,0\,\rangle\langle\,0\,|\,a_{k'}a_k + \text{H.c.}\big)\end{aligned}$$

Partial Trace: $\text{Tr}_{k'}(\rho_{kk'})$

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The fermionic ambiguity

Partial traces appear ambiguous

$$\mathrm{Tr}_{k'}(\rho_{kk'}) = (\alpha_1 + \alpha_2)\,|0\rangle\langle 0| + (\alpha_3 + \alpha_4)\,|1_k\rangle\langle 1_k| + \left[(\beta_2 \pm \beta_5)\,|0\rangle\langle 1_k| + \text{H.c}\right]$$

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Signs are fixed if a particular mapping to tensor product is chosen

But not in the right way!

Consistency conditions[1]

Expectation values of local observables invariant

$$\langle\,\mathcal{O}(k)\,\rangle_{\rho_{kk'}} = \langle\,\mathcal{O}(k)\,\rangle_{\mathrm{Tr}_{k'}(\rho_{kk'})}$$

[1] Friis, Lee, Bruschi, *Fermionic mode entanglement in quantum information*, Phys. Rev. A **87**, 022338 (2013).

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Consistency conditions determine signs uniquely[1]

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Consistent mappings to qubits (?)

Practical consequence of consistency conditions

Partial tracing "inside-out", e.g., Tr$_k$: $a_k^\dagger\,|0\rangle\langle 0|\,a_{k'}a_k$

anticommute towards vacuum projector

remove operators

Isomorphisms between n fermionic modes and n qubits

mappings from $\mathcal{F}_N$ to $\mathcal{H}_1 \otimes \ldots \otimes \mathcal{H}_N$

e.g., $a_k^\dagger\,|0\rangle\langle 0|\,a_{k'}a_k \;\mapsto\; |1\rangle\langle 1|_k \otimes |0\rangle\langle 1|_{k'}$

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Role of superselection rules

In general consistent mapping should satisfy

$$\begin{array}{ccc} \rho_{AB} \in \mathcal{F}_{N_{AB}} & \longleftrightarrow & \tilde{\rho}_{AB} \in \mathcal{H}_A \otimes \mathcal{H}_B \\ \downarrow & & \downarrow \\ \rho_A \in \mathcal{F}_{N_A} & \longleftrightarrow & \tilde{\rho}_A \in \mathcal{H}_A \end{array}$$

Can this be fixed by superselection rules?

Superselection rules restrict allowed density operator elements

2 modes: Yes, this is enough, problematic elements disappear

3 modes: No, does not work in general (for counterexample see [1])

[1] Friis, Lee, Bruschi, Fermionic mode entanglement in quantum information, Phys. Rev. A 87, 022338 (2013)

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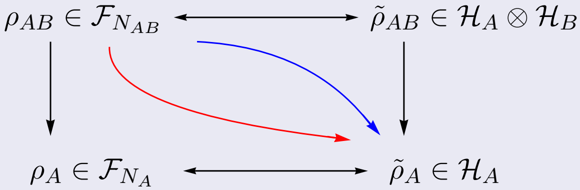

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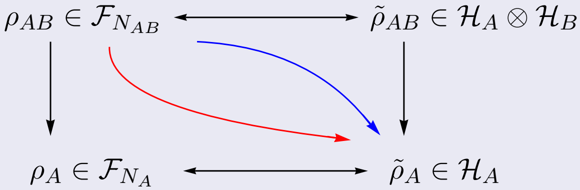
In general consistent mapping should satisfy



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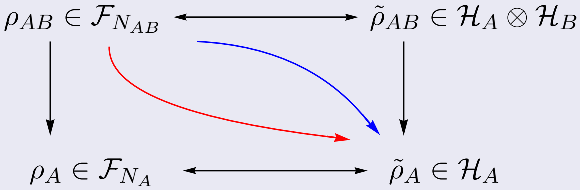

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So what are the superselection rules good for?

Suppose there were no superselection rules...

pure state of two modes k and k' prime:

$$|\psi\rangle = \mu_0 |0\rangle + \mu_k |1_k\rangle + \mu_{k'} |1_{k'}\rangle + \mu_{kk'} |1_k, 1_{k'}\rangle$$

reduced states:

$$\rho_k = (|\mu_0|^2 + |\mu_{k'}|^2) |0\rangle\langle 0| + (|\mu_k|^2 + |\mu_{kk'}|^2) |1_k\rangle\langle 1_k| + \big[(\mu_0\mu_k^* + \mu_{k'}\mu_{kk'}^*) |0\rangle\langle 1_k| + H.c.\big]$$

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e.g., for $\mu_0 = \mu_k = \mu_{k'} = \mu_{kk'} = \frac{1}{2}$: ρ_k pure, $\rho_{k'}$ maximally mixed

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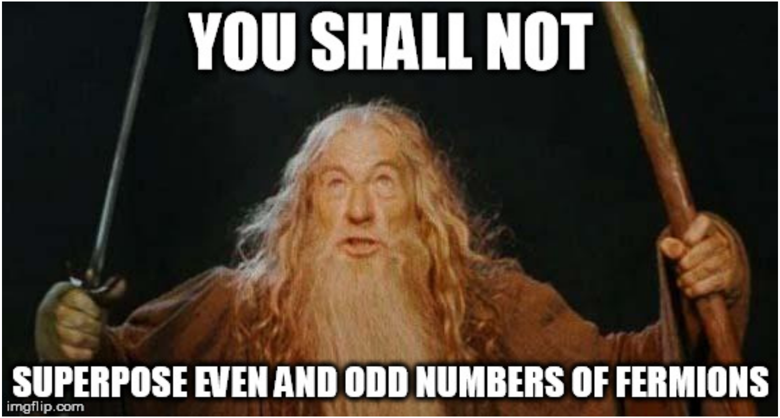
The Parity Superselection Rule



Parity superselection prevents asymmetric pure states

Sketch of proof[2]

set of n fermionic modes $N = \{1, 2, \ldots, n\}$ & pure state $|\psi^N_{\text{even}}\rangle$

bipartition: $M = \{\mu_i \in N | i = 1, \ldots, m < n\}$ and $M^C = N/M$

write state as (w.l.o.g. m is here odd)

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Origin of the parity superselection rule

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- "I don't like asymmetric pure state marginals."

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  Practical problem, not fundamental + $\exists$ neutral fermions

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Implications for fermions in quantum information

Quantum information in fermionic Fock space

- Density operators well defined
- Reduced states well defined (consistency conditions + SSR!)
- Entanglement ⇔ mixedness of subsystems
- Entanglement of formation well defined (computable? [4])

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[6] Dasenbrook, Bowles, Bohr Brask, Hofer, Flindt, Brunner, e-print arXiv:1511.04450 [cond-mat.mes-hall] (2015).

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Quantum information in fermionic Fock space

- Density operators well defined
- Reduced states well defined (consistency conditions + SSR!)
- Entanglement $\Leftrightarrow$ mixedness of subsystems
- Entanglement of formation well defined (computable? [4])

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- Computable criteria, e.g., PPT, rely on tensor product
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Conclusion

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