

# Quantum information tasks with relativistically rigid cavities in non-uniform motion

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Engineering and Physical Sciences  
Research Council



The University of  
Nottingham

# Relativistic Quantum Information

## Motivation

- Resources/Tasks of QI well known:

How are they affected by *Relativity?* degraded/enhanced?

- Connection between *QFT*  $\Rightarrow$  *QI*

*Unruh effect, Hawking effect, Casimir effect*

- New Ways to ... create, store, transmit, process QI

- Possibly new directions/hints towards quantum gravity?

Our aim here ...

Effects of *Non-uniform Motion* on Entanglement

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# Outline

Express to Alpha Centauri ... now boarding

- Non-uniform cavity motion:  
construction of generic trajectories
- Bogoliubov transformations and entanglement  
Comparison of procedures for Fock states and Gaussian states
- teleportation with non-uniformly moving cavities  
simulation with superconducting circuits

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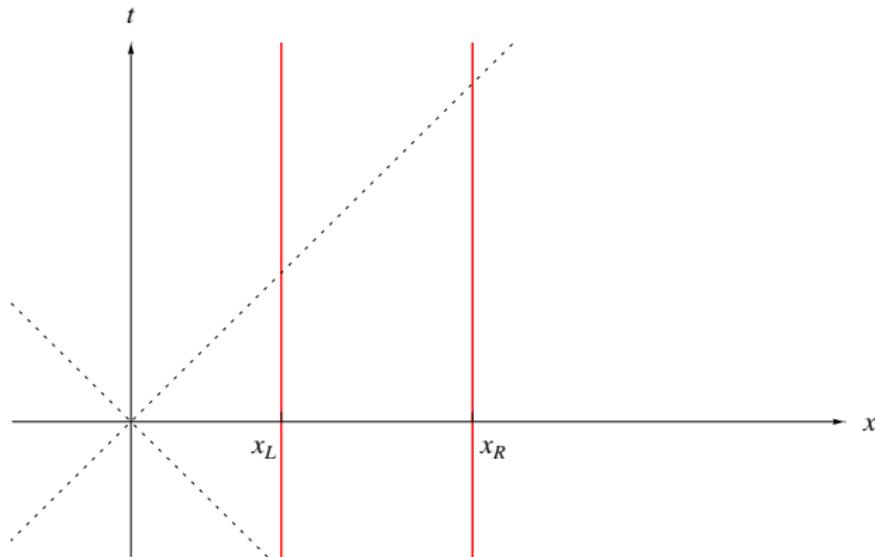
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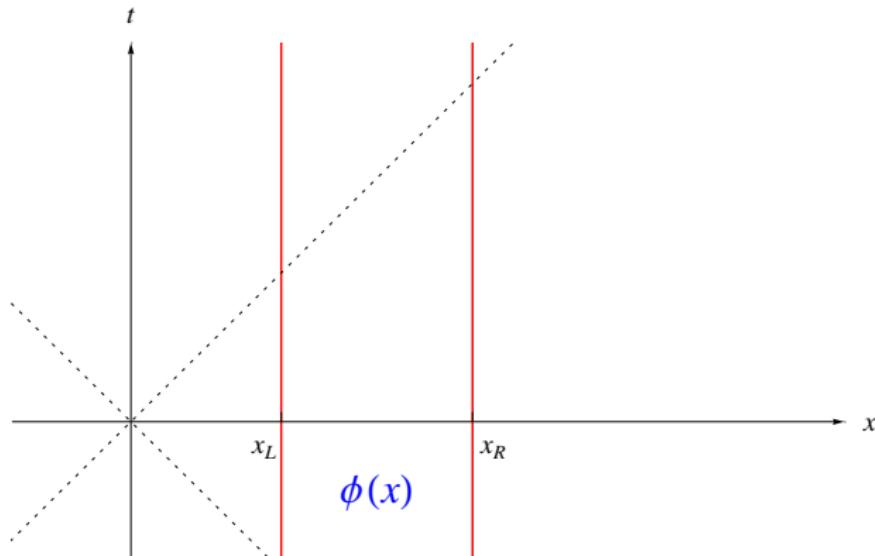
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# Inertial cavity of width $\delta = b - a$



Klein-Gordon Equation:  $(\partial^2/\partial t^2 - m^2) \phi(x,t) = 0$

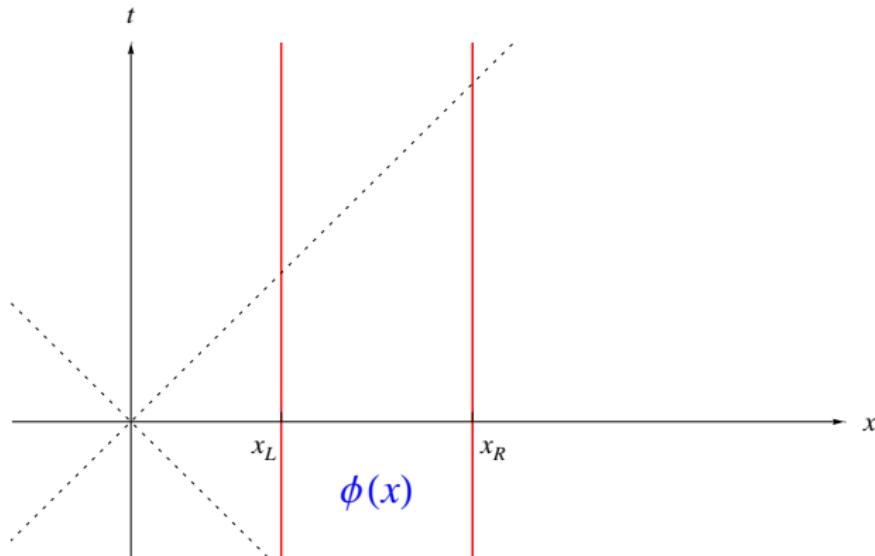
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Uncharged scalar field  $\phi$

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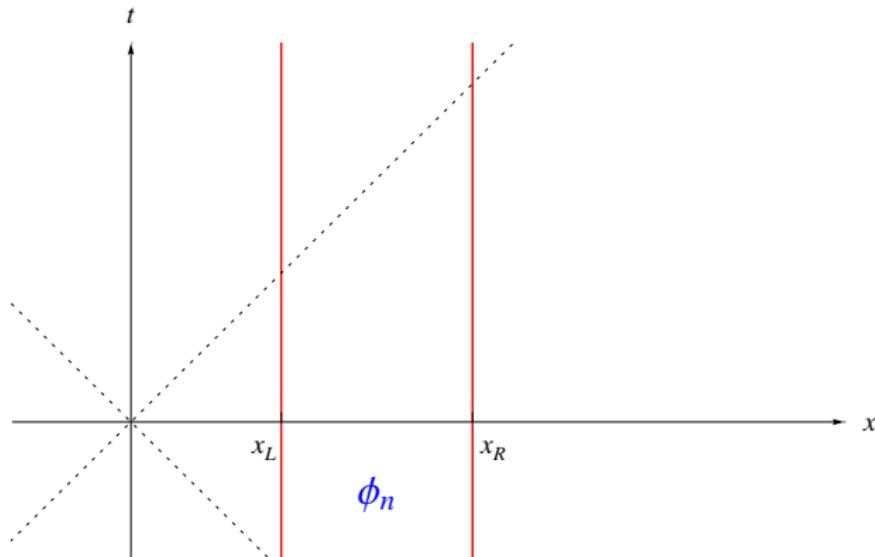
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Uncharged scalar field  $\phi$

boundary conditions: *Dirichlet*     $\phi(t, \mathbf{a}) = \phi(t, \mathbf{b}) = 0$

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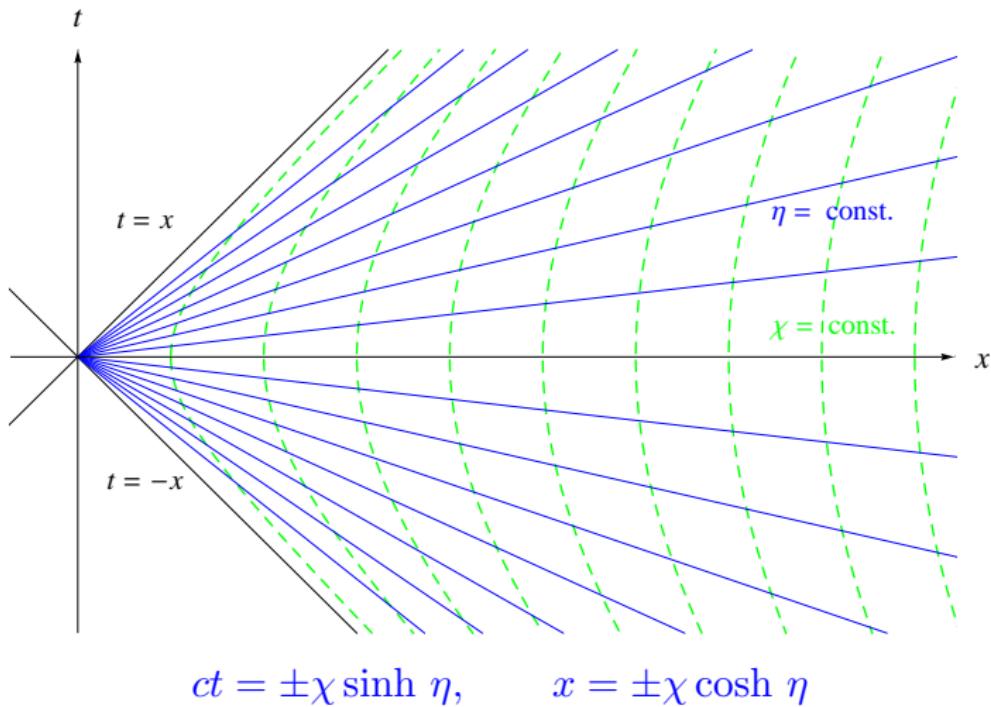


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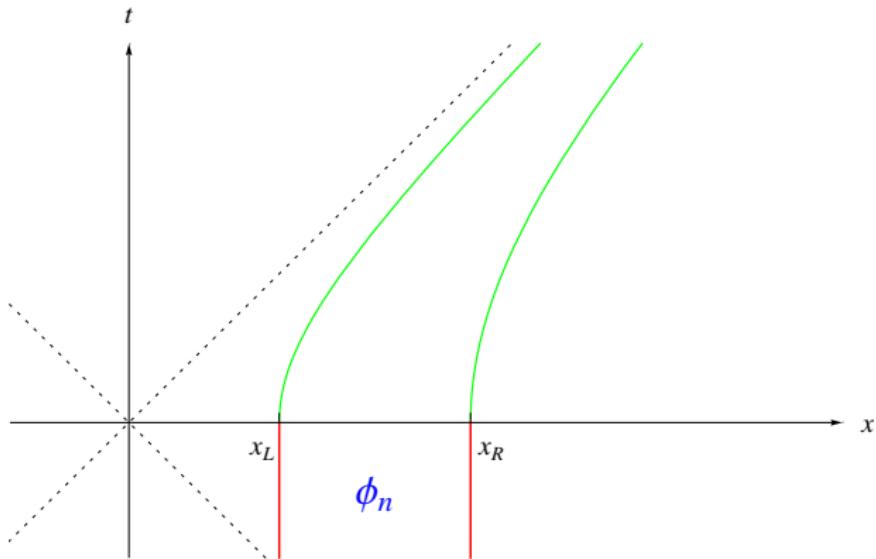
solutions:  $\{\phi_n\}$ :  $\phi = \sum_n (\phi_n a_n + \phi_n^* a_n^\dagger)$ ,  $[a_m, a_n^\dagger] = \delta_{mn}$

# Uniform acceleration in Minkowski spacetime

Rindler coordinates  $(\eta, \chi)$

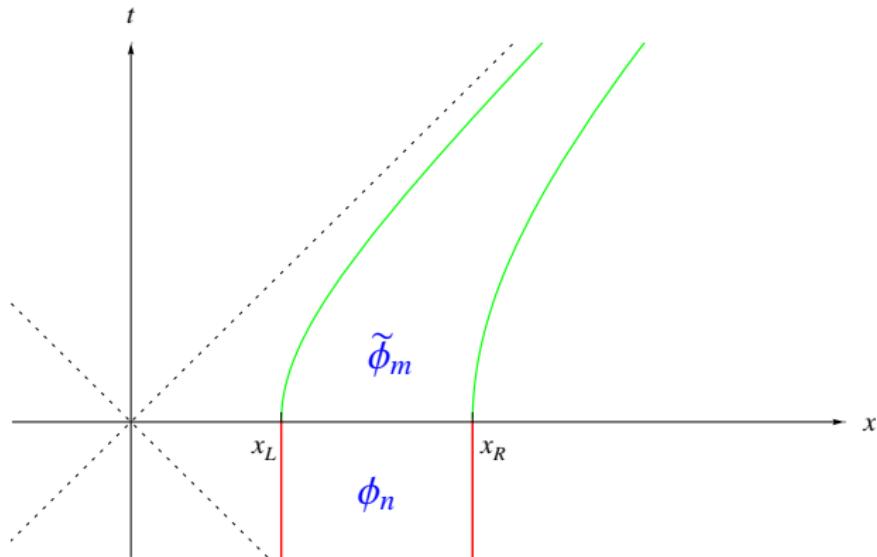


# Quantum fields in accelerated cavities



Repeat quantisation procedure in Rindler coordinates

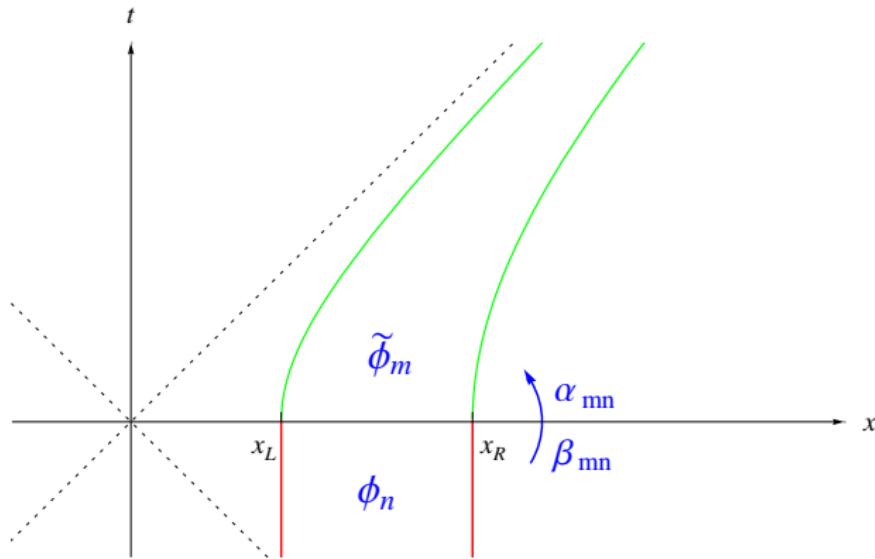
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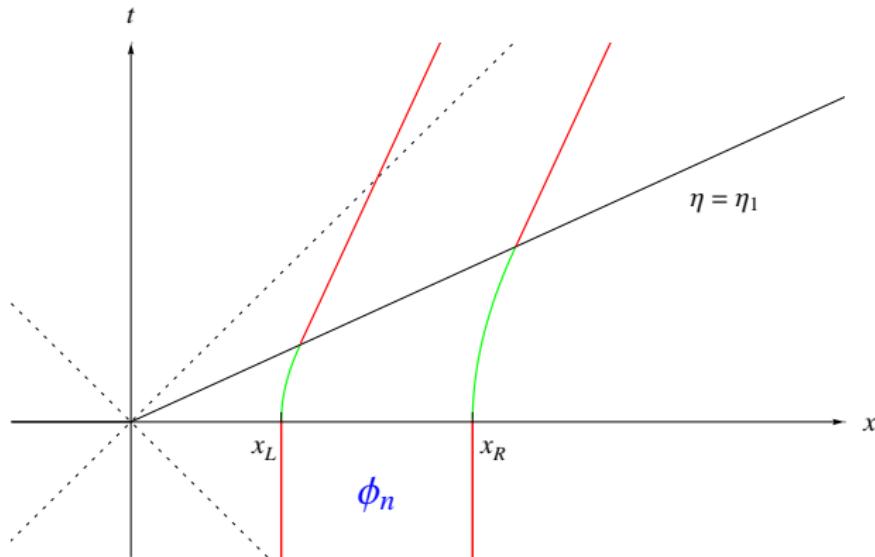
# Quantum fields in accelerated cavities



Solutions related by Bogoliubov transformation:

$$\phi_n = \sum_m (\alpha_{mn}^* \tilde{\phi}_m - \beta_{mn} \tilde{\phi}_m^*), \quad \alpha_{mn} = (\tilde{\phi}_m, \phi_n), \quad \beta_{mn} = -(\tilde{\phi}_m, \phi_n^*)$$

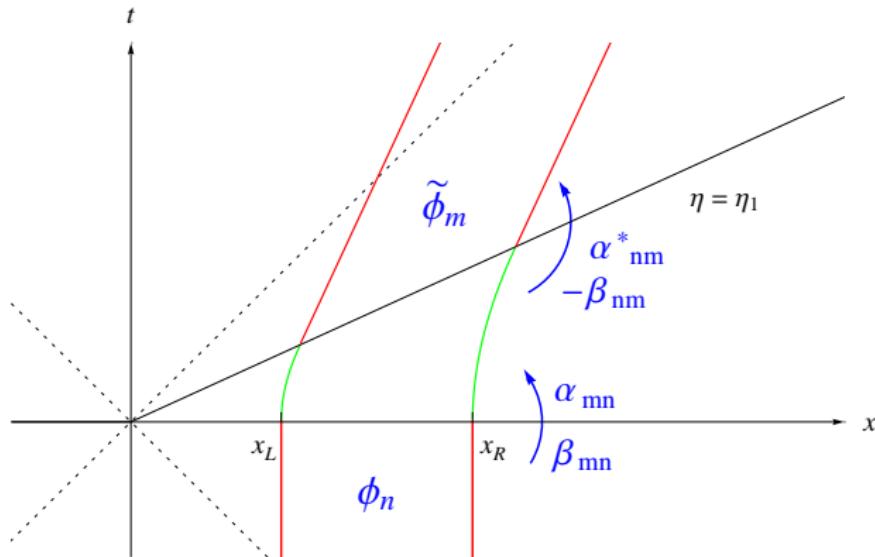
# Quantum fields in accelerated cavities



Finite duration of acceleration:

acceleration stops at **fixed coordinate time**  $\eta = \eta_1$

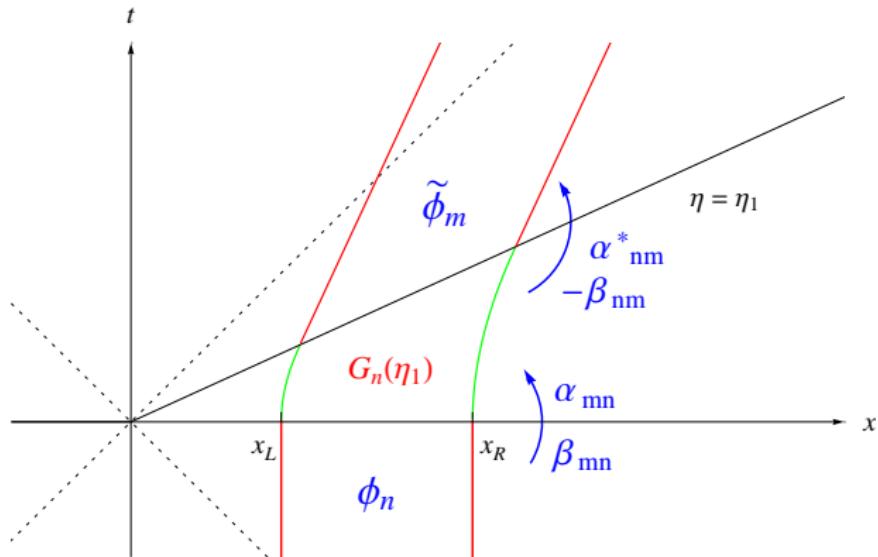
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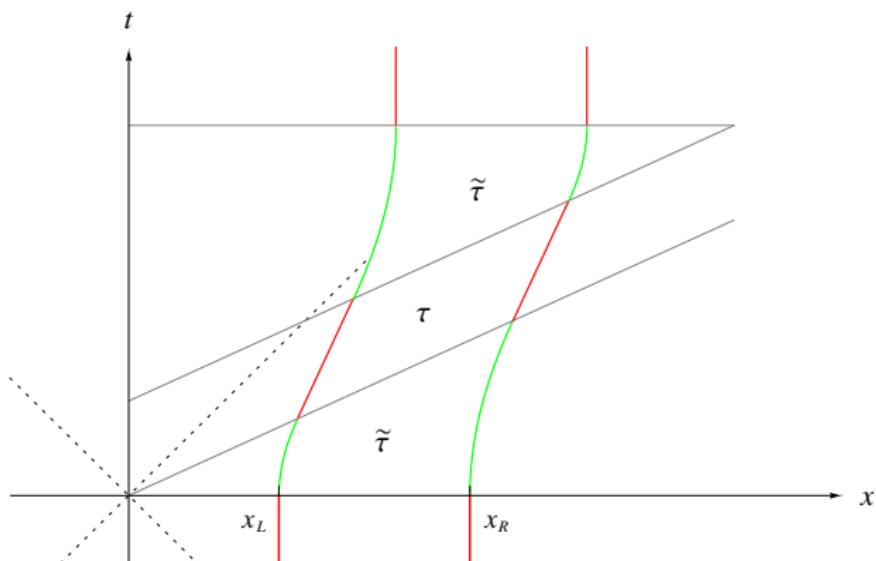
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Finite duration of acceleration:

modes pick up phases  $G_n(\eta_1)$

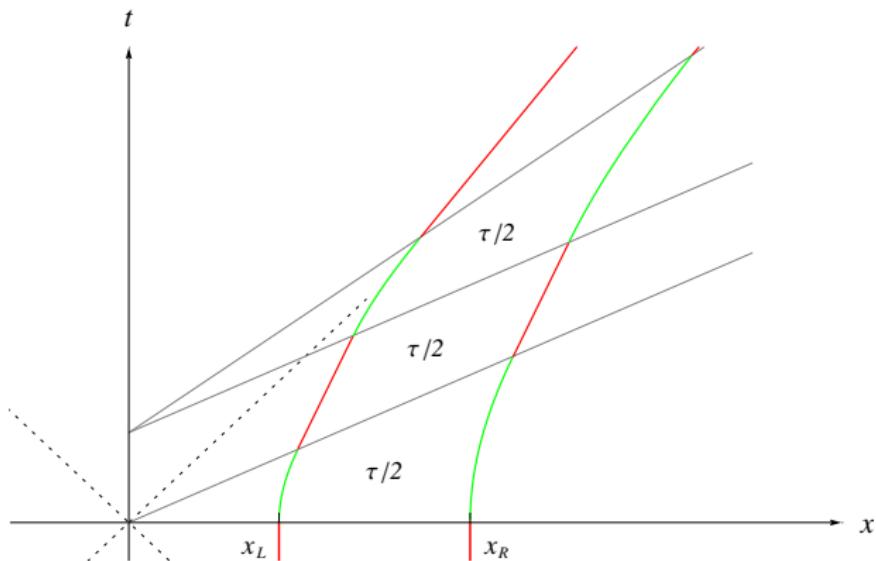
# Constructing generic travel scenarios



Building blocks can be freely combined: "Lego" system

Sample travel scenario: "Voyage to Alpha Centauri"

# Constructing generic travel scenarios



Building blocks can be freely combined: “Lego” system

Sample travel scenario: **consecutive boosts**

# Bogoliubov Transformations

(real, scalar) Bosonic quantum field  $\phi$  with *discrete spectrum*:

$$\phi = \sum_n (a_n \phi_n + a_n^\dagger \phi_n^*)$$

Bogoliubov transformation: “In-Region”  $\Leftrightarrow$  “Out-Region”

mode functions:  $\phi_n = \sum_m (\alpha_{mn}^* \tilde{\phi}_m - \beta_{mn} \tilde{\phi}_m^*)$

operators:  $a_n^\dagger = \sum_m (\alpha_{mn}^* \tilde{a}_m^\dagger + \beta_{mn} \tilde{a}_m)$

coefficients:  $\alpha_{mn} = (\tilde{\phi}_m, \phi_n), \quad \beta_{mn} = -(\tilde{\phi}_m, \phi_n^*)$

perturbative expansion: parameter  $h := 2 \frac{x_R - x_L}{x_R + x_L} = \mathbf{a}_c L$

$\alpha_{mn} = \alpha_{mn}^{(0)} + \alpha_{mn}^{(1)} h + O(h^2), \quad \beta_{mn} = \beta_{mn}^{(1)} h + O(h^2)$

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# Studying Entanglement in Fock space

## Transformation of Fock states

Vacuum:  $|0\rangle = N \exp\left(\frac{1}{2} \sum_{p,q} V_{pq} \tilde{a}_p^\dagger \tilde{a}_q^\dagger\right) |\tilde{0}\rangle$  with  $V = -\beta^* \alpha^{-1}$

unitarity: *Hilbert-Schmidt condition*  $\sum_{p,q} |V_{pq}|^2 < \infty$

particle states: act with  $a_n^\dagger = \sum_m \left( \alpha_{mn}^* \tilde{a}_m^\dagger + \beta_{mn} \tilde{a}_m \right)$  on  $|0\rangle$

## Usual Procedure

- Select in-region state
- Transform to out-region
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# Bogoliubov transformations in phase space

## Gaussian States - covariance matrix formalism

$$\sigma_{ij} = \langle \mathbb{X}_i \mathbb{X}_j + \mathbb{X}_j \mathbb{X}_i \rangle - 2 \langle \mathbb{X}_i \rangle \langle \mathbb{X}_j \rangle$$

quadratures:  $\mathbb{X}_{(2n-1)} = \frac{1}{\sqrt{2}}(a_n + a_n^\dagger)$  and  $\mathbb{X}_{(2n)} = \frac{-i}{\sqrt{2}}(a_n - a_n^\dagger)$

Bogoliubov transformation  $\Rightarrow$  symplectic transformation  $S$

$$\tilde{\sigma} = S \sigma S^T$$

$$S = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} & \dots \\ \mathcal{M}_{21} & \mathcal{M}_{22} & \mathcal{M}_{23} & \dots \\ \mathcal{M}_{31} & \mathcal{M}_{32} & \mathcal{M}_{33} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \text{ where } \mathcal{M}_{mn} = \begin{pmatrix} \text{Re}(\alpha_{mn} - \beta_{mn}) & \text{Im}(\alpha_{mn} + \beta_{mn}) \\ -\text{Im}(\alpha_{mn} - \beta_{mn}) & \text{Re}(\alpha_{mn} + \beta_{mn}) \end{pmatrix}$$

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$$S = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} & \cdots \\ \mathcal{M}_{21} & \mathcal{M}_{22} & \mathcal{M}_{23} & \cdots \\ \mathcal{M}_{31} & \mathcal{M}_{32} & \mathcal{M}_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \text{ where } \mathcal{M}_{mn} = \begin{pmatrix} \text{Re}(\alpha_{mn} - \beta_{mn}) & \text{Im}(\alpha_{mn} + \beta_{mn}) \\ -\text{Im}(\alpha_{mn} - \beta_{mn}) & \text{Re}(\alpha_{mn} + \beta_{mn}) \end{pmatrix}$$

# Tracing & Entanglement

## Transforming initial state

$$\sigma = \begin{pmatrix} C_{11} & C_{12} & C_{13} & \dots \\ C_{21} & C_{22} & C_{23} & \dots \\ C_{31} & C_{32} & C_{33} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \Rightarrow \tilde{\sigma} = \begin{pmatrix} \tilde{C}_{11} & \tilde{C}_{12} & \tilde{C}_{13} & \dots \\ \tilde{C}_{21} & \tilde{C}_{22} & \tilde{C}_{23} & \dots \\ \tilde{C}_{31} & \tilde{C}_{32} & \tilde{C}_{33} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where  $\tilde{C}_{mn} = \sum_{i,j} \mathcal{M}_{mi} C_{ij} \mathcal{M}_{nj}^T$

• Tracing out one cavity leads to a mixed state  
 • Entanglement between cavities

# Tracing & Entanglement

Transforming initial state

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Compute entanglement measure of choice

e.g., (logarithmic) Negativity  $\mathcal{N}$ , Gaussian Contangle,  
 Entropy of Entanglement, etc.

# Tracing & Entanglement

Tracing out inaccessible modes, e.g., all but 2 & 3

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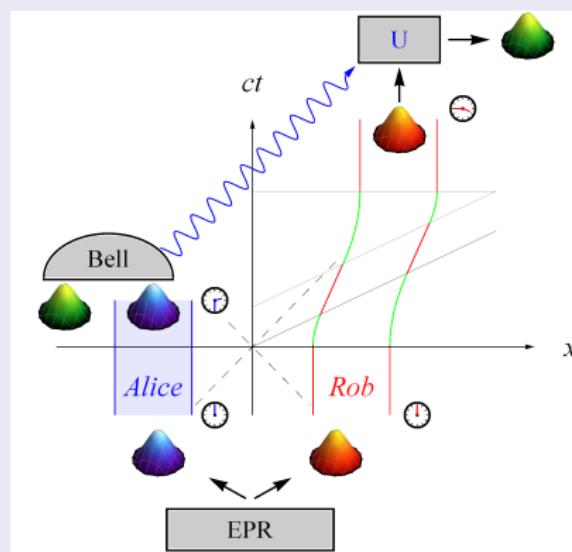
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## Results - Degradation of Entanglement

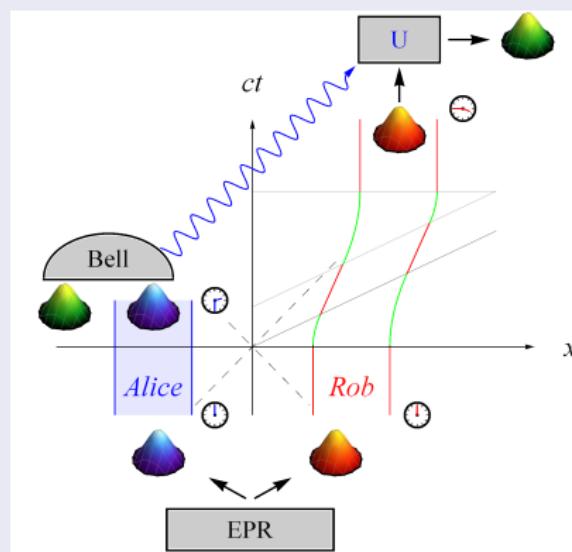
"in-region": 2-mode squeezed state of *Alice's mode k* and *Rob's mode k'*



"out-region": Trace out all other modes: entanglement degradation

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N. Friis, A. R. Lee, K. Truong, C. Sabín, E. Solano, G. Johansson and I. Fuentes , Phys. Rev. Lett. **110**, 113602 (2013).

## Results - Degradation of Entanglement

“in-region”: 2-mode squeezed state of *Alice’s mode  $k$*  and *Rob’s mode  $k'$*

$$\text{Teleportation Fidelity: } \tilde{\mathcal{F}} = \tilde{\mathcal{F}}^{(0)} - \tilde{\mathcal{F}}^{(2)} h^2 + \mathcal{O}(h^4)$$

where

$$\tilde{\mathcal{F}}^{(0)} = (1 + \text{Cosh}(2r) - \text{Cos}(\phi) \text{Sinh}(2r))^{-1},$$

$$\tilde{\mathcal{F}}^{(2)} = (\tilde{\mathcal{F}}^{(0)})^2 (1 + e^{-2r}) (f_{k'}^\alpha + f_{k'}^\beta \text{Tanh}(2r))$$

$$\text{with } f_{k'}^\alpha = \sum_n |\alpha_{nk'}^{(1)}|^2 \text{ and } f_{k'}^\beta = \sum_n |\beta_{nk'}^{(1)}|^2$$

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## Results - Degradation of Entanglement

"in-region": 2-mode squeezed state of *Alice's mode k* and *Rob's mode k'*

$$\text{Teleportation Fidelity: } \tilde{\mathcal{F}}_{\text{opt}} = \tilde{\mathcal{F}}_{\text{opt}}^{(0)} - \tilde{\mathcal{F}}_{\text{opt}}^{(2)} h^2 + \mathcal{O}(h^4)$$

where

$$\tilde{\mathcal{F}}_{\text{opt}}^{(0)} = (1 + e^{-2r})^{-1},$$

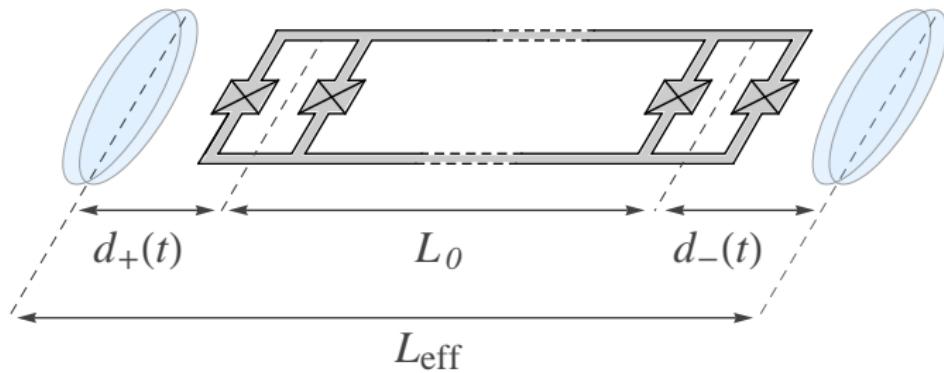
$$\tilde{\mathcal{F}}_{\text{opt}}^{(2)} = \tilde{\mathcal{F}}_{\text{opt}}^{(0)} (f_{k'}^\beta + f_{k'}^\alpha \tanh(2r))$$

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# Experimental Proposal

$1 - \dim$  transmission line for microwave radiation



Superconducting circuits (SQUIDS) simulate boundary conditions

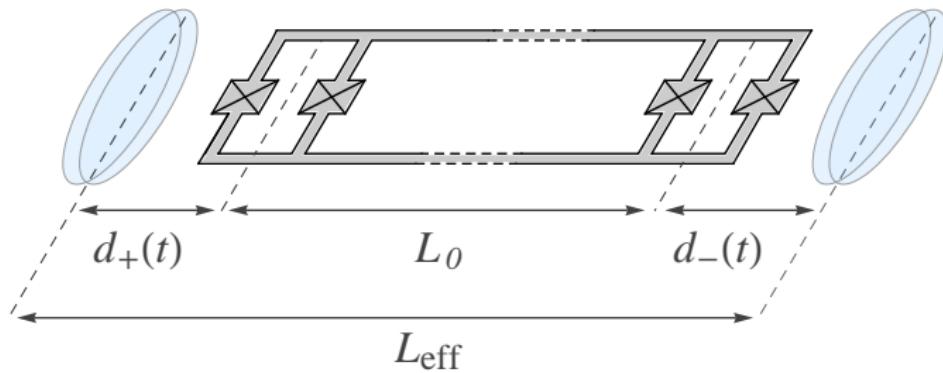
$$\Rightarrow \text{cavity of effective length } L_{\text{eff}} = L_0 + d_+(t) + d_-(t)$$

predicted degradation of optimal teleportation fidelity

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predicted degradation of optimal teleportation fidelity:  $\approx 4\%$

# Thank you for your attention.

## Further reading:

**recent RQI review:** P. M. Alsing and I. Fuentes, Classical Quantum Gravity **29**, 224001 (2012) [arXiv:1210.2223].  
**Detector models in RQI:** A. R. Lee, Ph.D. thesis, University of Nottingham, 2013 [arXiv:1309.4419].

## Rigid cavities in RQI:

**full monty:** N. F., Ph.D. thesis, University of Nottingham, 2013 [on the arXiv soon].

**initial paper:** D. E. Bruschi, I. Fuentes, and J. Louko, Phys. Rev. D **85**, 061701(R) (2012) [arXiv:1105.1875].

**fermions:** N. F., A. R. Lee, D. E. Bruschi, and J. Louko, Phys. Rev. D **85**, 025012 (2012) [arXiv:1110.6756].

**entanglement generation:** N. Friis, D. E. Bruschi, J. Louko, and I. Fuentes, Phys. Rev. D **85**, 081701(R) (2012) [arXiv:1201.0549].

**entanglement resonances:** D. E. Bruschi, A. Dragan, A. R. Lee, I. Fuentes, and J. Louko, Phys. Rev. Lett. **111**, 090504 (2013) [arXiv:1201.0663].

**multipartite entanglement:** N. Friis, M. Huber, I. Fuentes, and D. E. Bruschi, Phys. Rev. D **86**, 105003 (2012) [arXiv:1207.1827].

**Gaussian states:** N. Friis and I. Fuentes, J. Mod. Opt. **60**, 22 (2013) [arXiv:1204.0617].

**teleportation & simulation:** N. Friis, A. R. Lee, K. Truong, C. Sabín, E. Solano, G. Johansson, and I. Fuentes, Phys. Rev. Lett. **110**, 113602 (2013) [arXiv:1211.5563].

**smooth accelerations:** D. E. Bruschi, J. Louko, D. Faccio, and I. Fuentes, New J. Phys. **15**, 073052 (2013) [arXiv:1210.6772].

**unitarity & photons:** N. Friis, A. R. Lee, and J. Louko, Phys. Rev. D **88**, 064028 (2013) [arXiv:1307.1631].