Gaussian Units

- Class of easily implementable operations (generated by Hamiltonians at most quadratic in mode operators)
- Lowest in hierarchy of operations, require only two-body interactions
- Map all Gaussian states to Gaussian states

Gaussian states: quantum states of bosonic modes with Gaussian Wigner function
- Fully described by vector of first moments \( \langle X \rangle \) and \( \langle Y \rangle \) and second moments collected in covariance matrix \( \Gamma \) with components \( \Gamma_{ij} = \langle X_i X_j \rangle \) for any \( i,j \) and symplectic operators, e.g. for \( n^\text{th} \) mode:
- Symplectic form \( \omega(X,Y) = \sum_{i=1}^n dX_i dY_i \)

Gaussian unitaries: affine symplectic maps

Passivity vs. Gaussian Passivity

Passivity \( \Rightarrow \) Gaussian passivity \( \Rightarrow \) Gaussian passivity
For Gaussian initial states: Gaussian passivity
For general initial states: Gaussian passivity, uniquely defined by \( \chi \) and \( \Gamma \)

Theorem 1: For Gaussian states, Gaussian passivity is determined by the symplectic form \( \chi \)

Theorem 2: The 1st and 2nd moments of any Gaussian-passive state with entropy \( S_N \) are compatible with a (non-Gaussian) state \( \rho \) with \( S(\rho) \neq S_N \) and for which the maximal amount of energy (the energy difference to a thermal state with entropy \( S(\rho) \)) is extractable by general unitaries.

Gaussian Battery Charging

- Limitation of Gaussian Unitaries:
- Energy density is bounded on thermodynamic state space
- Bosonic marginals are Gaussian

Precision example: pure displacement \( |\Delta X| = |\Delta Y| = 1 \)
- For general Gaussian unitaries:
  - Optimal: combination of squeezing & displacement
  - Worst case: pure single mode squeezing

Fluctuations:
- Optimal combination of squeezing & displacement
- Worst case: in general combination of squeezing & displacement

Minimal Fluctuations

- For integer multiples of \( |\Delta X|, |\Delta Y| \) when \( \Delta X = m \in \mathbb{Z} \) shift by \( n_1 \) to the right
- For non-integer \( \Delta X \):
  - Start shifting at \( n_1 = \lceil \Delta X \rceil + 1 \)
  - Fine-tune rotation between \( n_1 \) and \( n_2 \)

Precision: Optimal and Worst Strategies

Single-mode batteries
- For \( T = 1 \), variance may decrease
- For fixed \( T \), \( \Delta \text{Var}(X) \) bounded by constants

Multi-mode batteries
- Already local unitaries provide advantage
- Correlations can occur during step II
- Correlations can help but play no central role

Variance

For Gaussian states, variance is Gaussian in \( n \)

References
- N. Friis and M. Huber, Quantum 2, 61 (2018) [arXiv:1708.00749]

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