Flexible Resources for Quantum Metrology

Workshop Secure Communication via Quantum Channels
Center for Interdisciplinary Research (ZIF), Bielefeld
26 April 2017

Nicolai Friis

Institute for Quantum Optics and Quantum Information – IQOQI Vienna

Work in collaboration with Davide Orsucci, Pavel Sekatski, Vedran Dunjko, Hans J. Briegel, Wolfgang Dür (University of Innsbruck) & Michalis Skotiniotis (Universitat Autònoma de Barcelona)
arXiv:1610.09999

Motivation

Quantum Metrology

Quantum Key Distribution

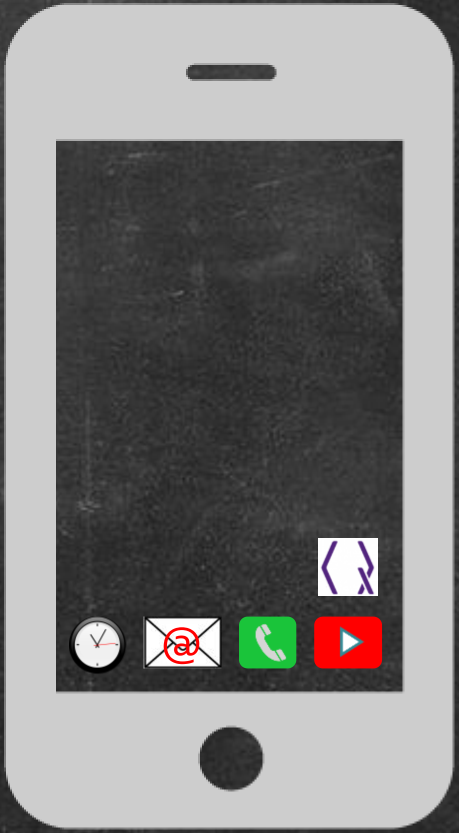
Quantum Communication

Quantum Computation

Quantum Games

Quantum Memories

VS



Motivation

We show: **efficient**, flexible implementation possible!

Quantum Metrology

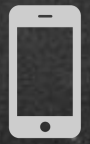
phase estimation
frequency estimation
Interaction strength estimation
Multi-parameter estimation
SU(2) estimation
etc.

&

"Local" estimation paradigm

vs.

Bayesian estimation paradigm

Desireable to have single architecture  for all of these tasks

Challenge: different tasks require vastly different probe states and measurements

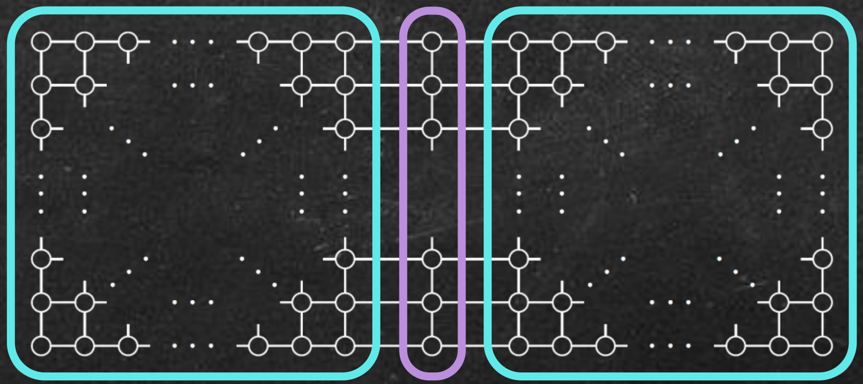
➡ **Solution: 2D Cluster States**

Universal for quantum computation

➡ Arbitrary preparation & measurement

MBQC ➡ Simple unifying framework

➡ Unambiguous resource cost: # qubits



Cost?

$O(N)$?

$O(N^2)$?

Local Estimation

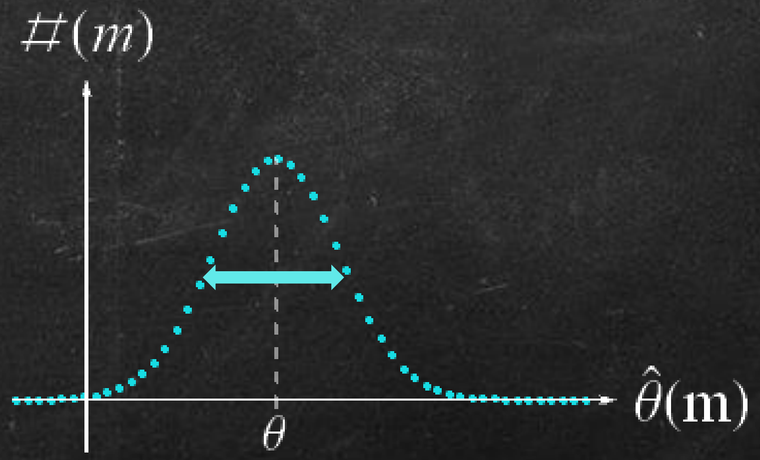
Parameter estimation problems

Quantum state		CPTP map		POVM $\sum_m E_m = \mathbb{1}$
ρ	$\rightarrow$	$\rho(\theta) = \mathcal{E}_\theta(\rho)$	$\rightarrow$	$\{E_m\} \rightarrow m$
Preparation		Encoding		Measurement

Estimator $\hat{\theta}(m)$ assigns "guess" for θ to outcome m

Accuracy $\longleftrightarrow$ unbiasedness

Precision $\longleftrightarrow$ Variance



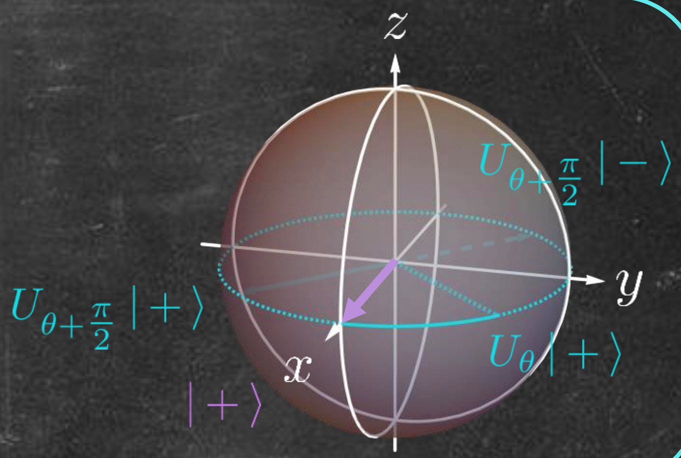
Local Phase Estimation

N-qubit phase estimation: $U_\theta = e^{-i\theta H}$ with $H = \sum_{i=1}^{N} H_i$ & $H_i = \frac{1}{2} Z \; \forall i$

Optimal classical strategy: Probe state $|\psi\rangle = |+\rangle^{\otimes N}$

Measurement $\{U_{\theta+\frac{\pi}{2}} |\pm\rangle\}$

Variance $V = \frac{1}{4N}$ "shot noise scaling"



Optimal quantum strategy: Probe state $|\psi_{\mathrm{GHZ}}\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + |1\rangle^{\otimes N})$

Measurement $M = X^{\otimes N}$ with $\langle M \rangle = \cos(N\theta)$ → $\hat{\theta}(m) = \frac{1}{N}\arccos(m)$

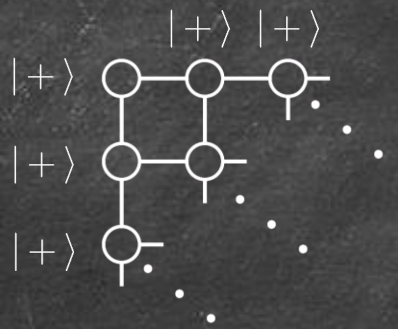
→ Variance $V[\hat{\theta}(m)] = (\Delta M)^2 \left(\frac{\partial \hat{\theta}}{\partial m}\right)^2 = \frac{1}{N^2}$ "Heisenberg scaling"

Implementation in Measurement-Based Quantum Computation (MBQC)

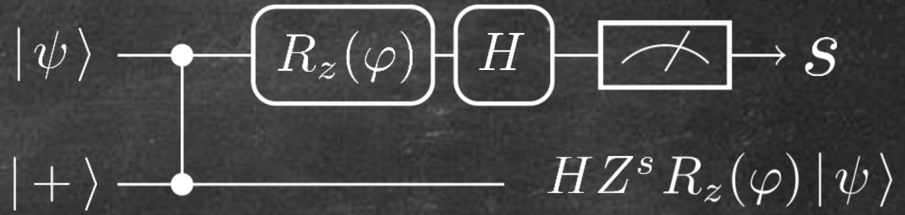
Cluster state:

Vertices: qubits in $|+\rangle$

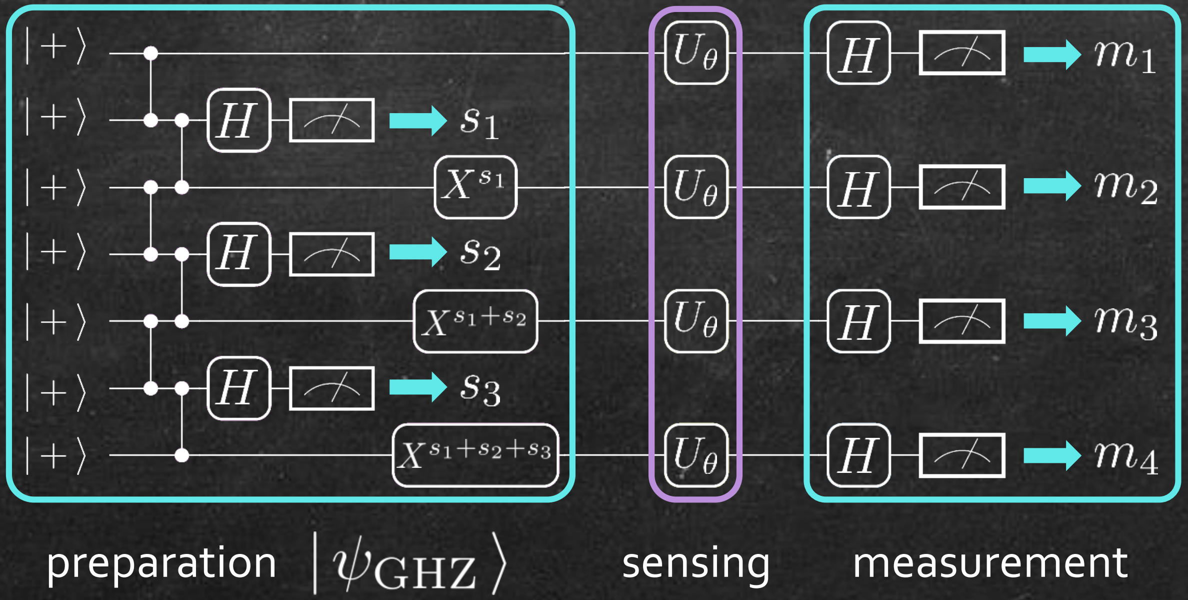
Edges: CZ gates



MBQC working principle: gate teleportation



Efficient Local Phase Estimation in MBQC



$$m = \prod_i m_i$$

$$\hat{\theta}(m) = \frac{1}{N}\arccos(m)$$

$$V[\hat{\theta}(m)] = \frac{1}{N^2}$$

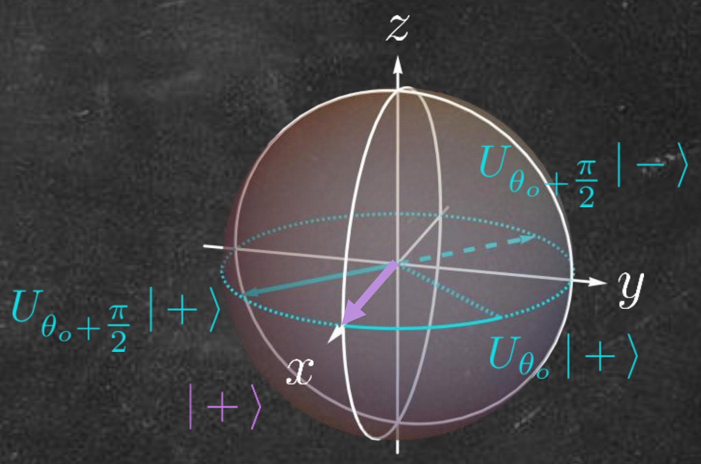
using $2N-1$ qubits

Bayesian Phase Estimation

Bayesian scenario: **prior** information $p(\theta)$ (e.g., Gaussian)

as before $p(m|\theta) = \mathrm{Tr}\big(E_m \rho(\theta)\big)$

but now $p(m) = \int d\theta \, p(m|\theta) \, p(\theta)$



Bayes' Law ➡ posterior $p(\theta|m) = \frac{p(m|\theta) \, p(\theta)}{p(m)}$

Conditional variance $V[\hat{\theta}(m)]$

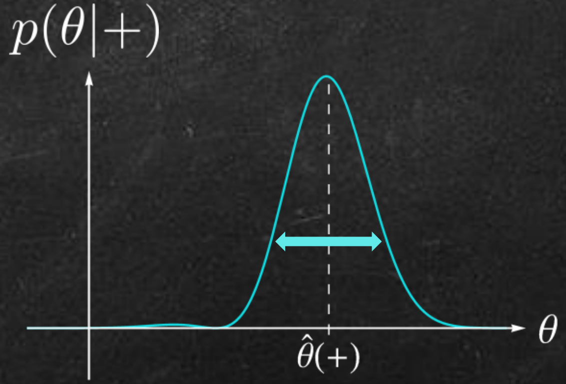


Figure of merit: **average variance** $\overline{V} = \sum_m p(m) \, V[\hat{\theta}(m)]$

Classical vs. Quantum Bayesian Estimation

Optimal classical strategy using N qubits

$|+\rangle^{\otimes N}$ & updating measurements

$\Rightarrow$ Bound $\overline{V} \geq \frac{1}{1/\sigma^2+N} \xrightarrow[N\to\infty]{} \frac{1}{N}$

(Gaussian priors)

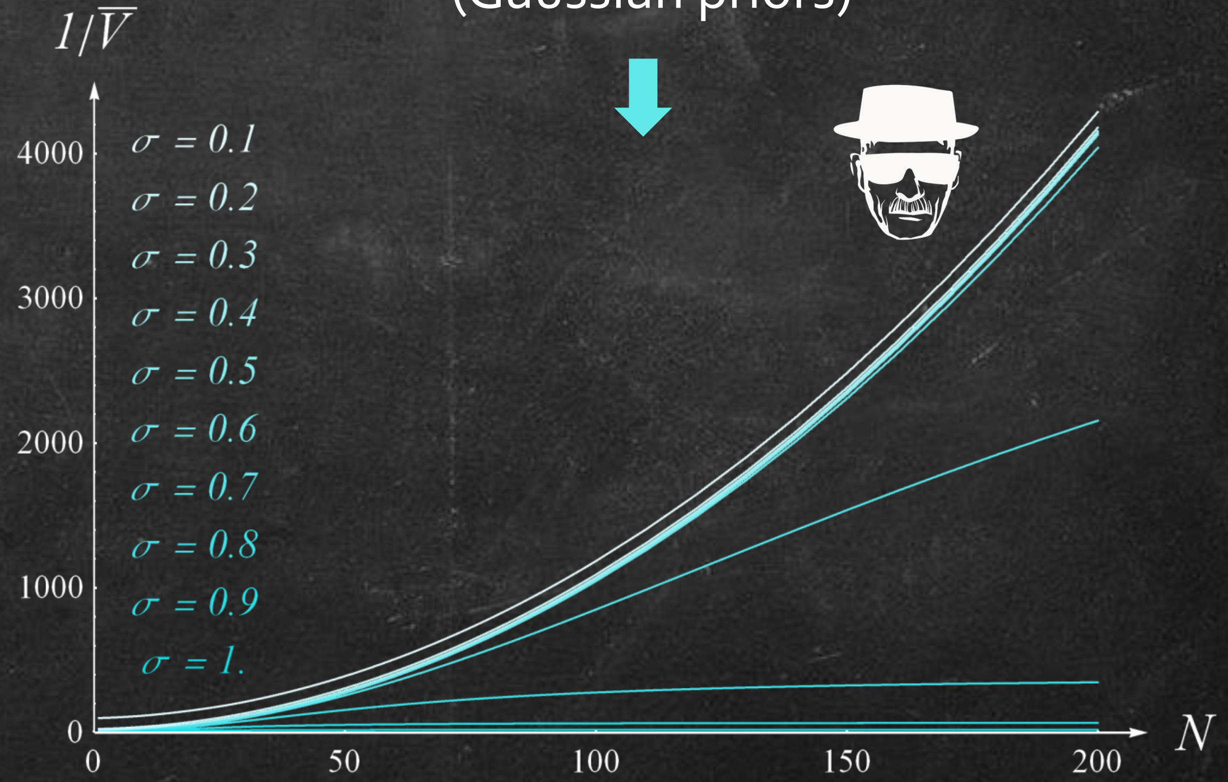
Quantum strategy

Superposition of $N+1$ eigenstates of H

$$|\psi\rangle = \sum_{n=0}^{N} \psi_n \, |n\rangle \quad \& \quad |n\rangle = |1\rangle^{\otimes n} \, |0\rangle^{N-n}$$

e.g., for $\psi_n = \sqrt{\frac{2}{N+2}} \sin\left(\frac{(n+1)\pi}{N+2}\right)$

discrete Fourier measurement $\left\{ |e_k\rangle = \frac{1}{\sqrt{N+1}} \sum_{n=0}^{N} e^{in\frac{2\pi k}{N+1}} \, |n\rangle \right\}$



Quantum Bayesian Estimation in MBQC ?

For any prior (and figure of merit) optimal probe can be chosen in $\mathcal{H}_\lambda = \text{span}\left\{ |\, n \,\rangle = |\, 1 \,\rangle^{\otimes n} \, |\, 0 \,\rangle^{N-n} \right\}$

Moreover $|\, \psi \,\rangle \in \mathcal{H}_\lambda$ ➡ $U_\theta \, |\, \psi \,\rangle \in \mathcal{H}_\lambda$ ➡ also measurements only in $\mathcal{H}_\lambda$

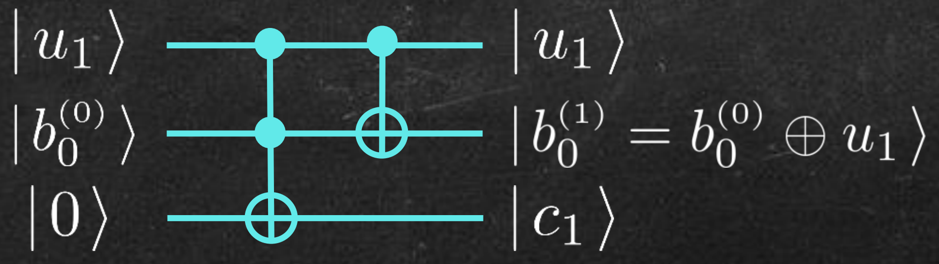
Note: $\dim(\mathcal{H}) = 2^N$ vs. $\dim(\mathcal{H}_\lambda) = N + 1$ ➡ $\lambda = \lceil \log_2(N+1) \rceil$ qubits

If $\exists$ efficient compression ➡ work in logarithmically small subspace

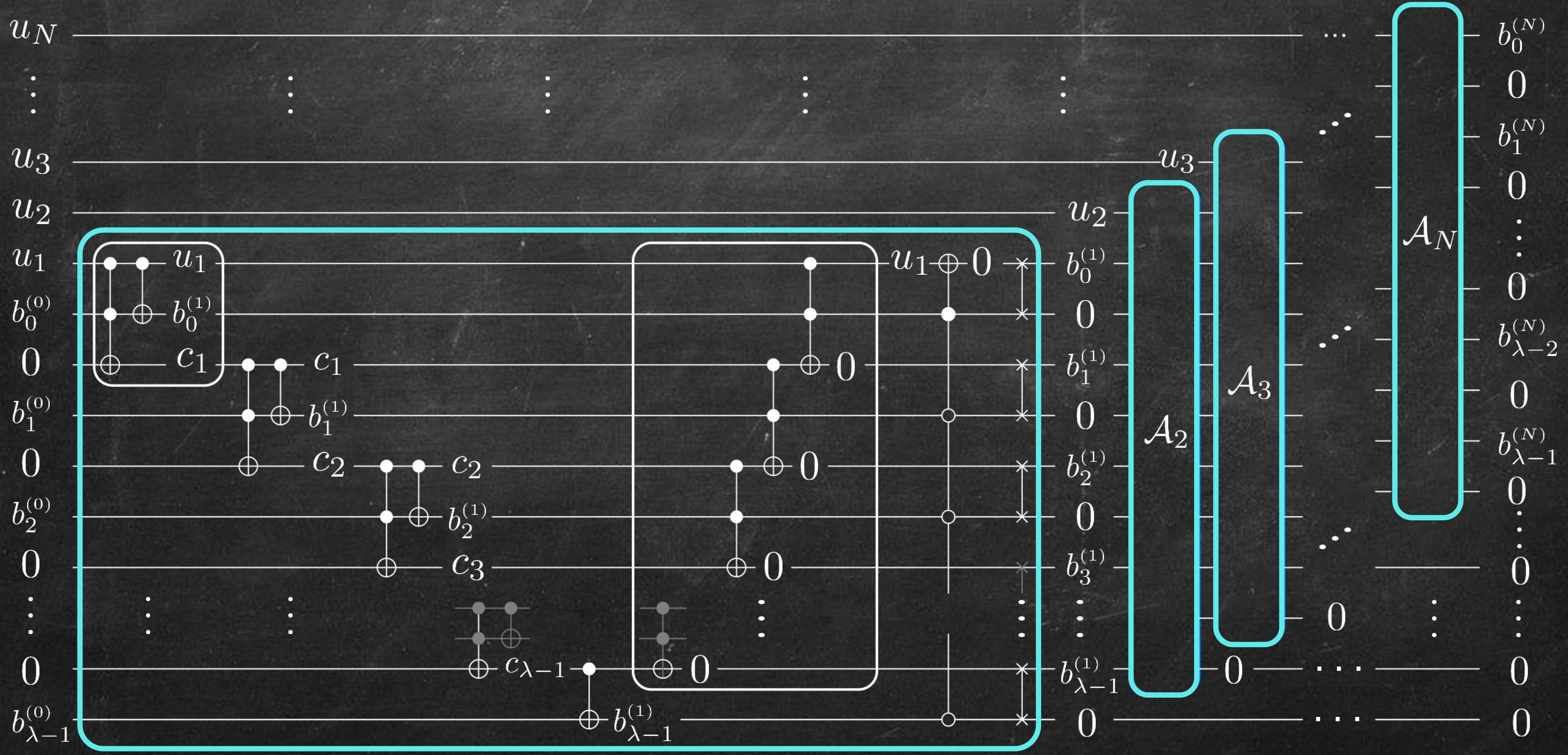
Convert unary representation $|\, n \,\rangle = |\, 1 \,\rangle^{\otimes n} \, |\, 0 \,\rangle^{N-n} = |\, u_1, u_2, \ldots, u_N \,\rangle$ with $u_i = 0, 1$

into binary representation $|\, n \,\rangle = |\, b_{\lambda-1}, b_{\lambda-2}, \ldots, b_1, b_0 \,\rangle$ with $b_i = 0, 1$

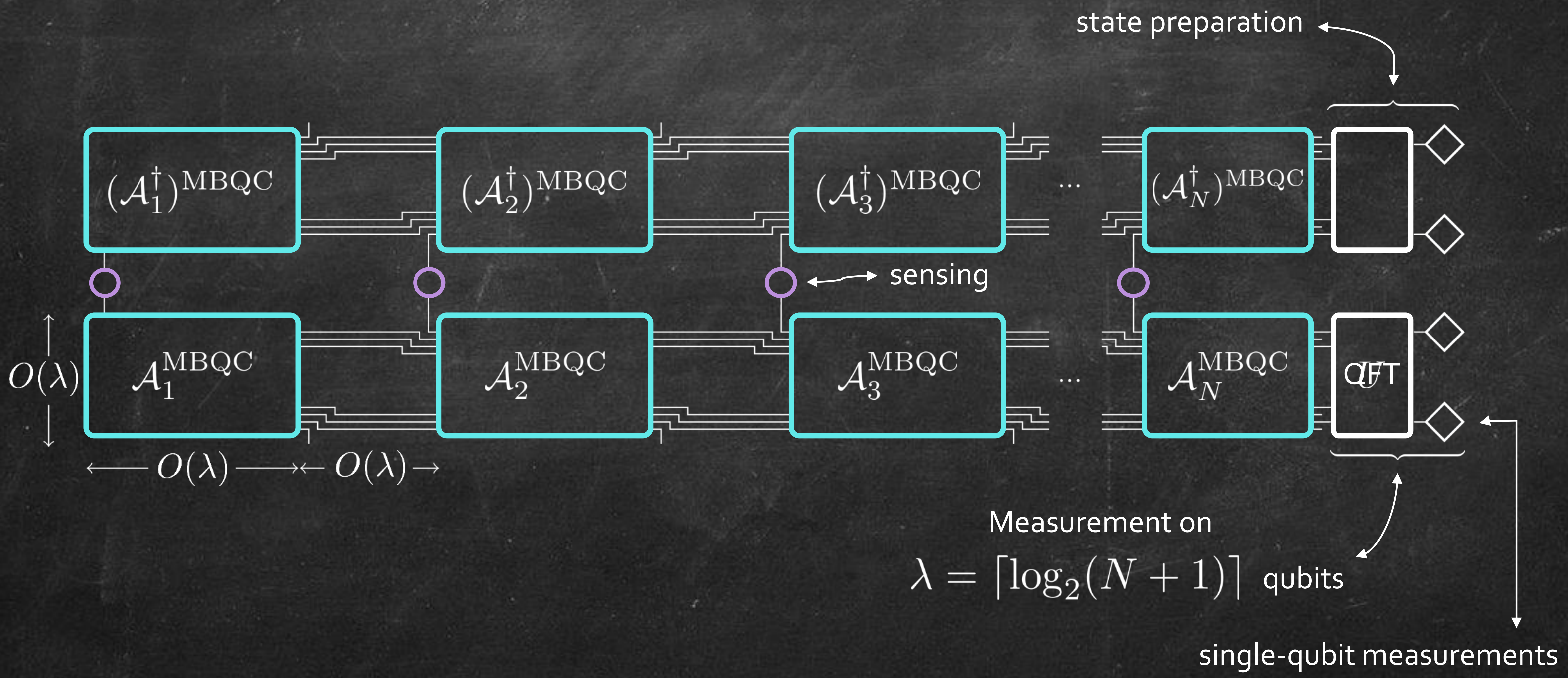
➡ Add qubit-by-qubit to empty register



Unary to Binary Compression



Quantum Bayesian Estimation in MBQC !

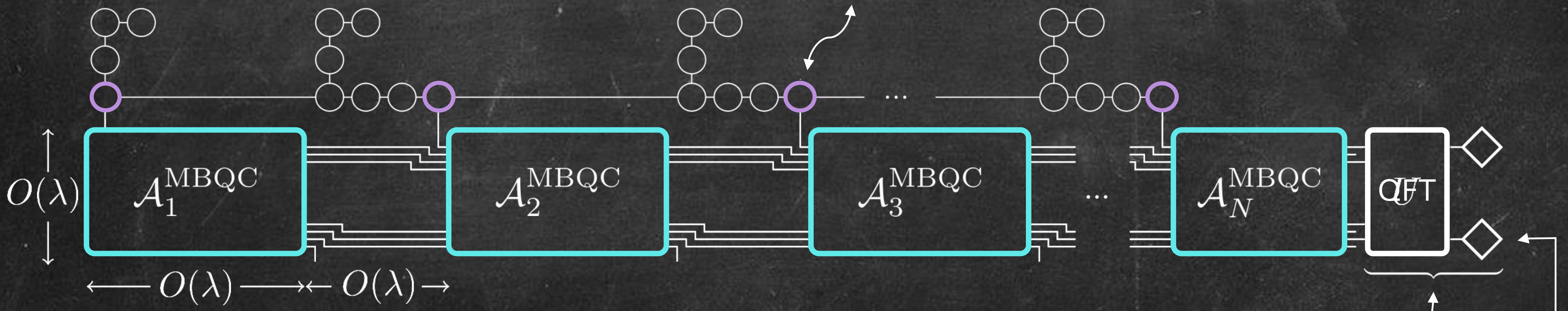


Quantum Bayesian Estimation in MBQC !

Overhead $O(N \log^2 N)$ ➡ Heisenberg scaling possible

state preparation

sensing



Measurement on

$\lambda = \lceil \log_2(N+1) \rceil$ qubits

Even if measurement (U) requires $O(2^\lambda)$ qubits

e.g., QFT requires $O(\lambda^2) = O(\log^2 N)$ qubits

single-qubit measurements

In conclusion:

- 2D Cluster states are flexible resources for quantum metrology
- In particular: local & Bayesian - phase & frequency estimation
- Also works for certain interaction Hamiltonians, e.g., Ising $H = \sum_{i,j} c_{ij} X_i \otimes X_j$
- Extensions to other estimation problems → work in progress

And thank you for your attention

Paper
arXiv:1610.09999

Thanks to my collaborators


Davide Orsucci


Pavel Sekatski


Vedran Dunjko


Wolfgang Dür


Hans J. Briegel


Michalis Skotiniotis