Implementing Quantum Control for Unknown Subroutines

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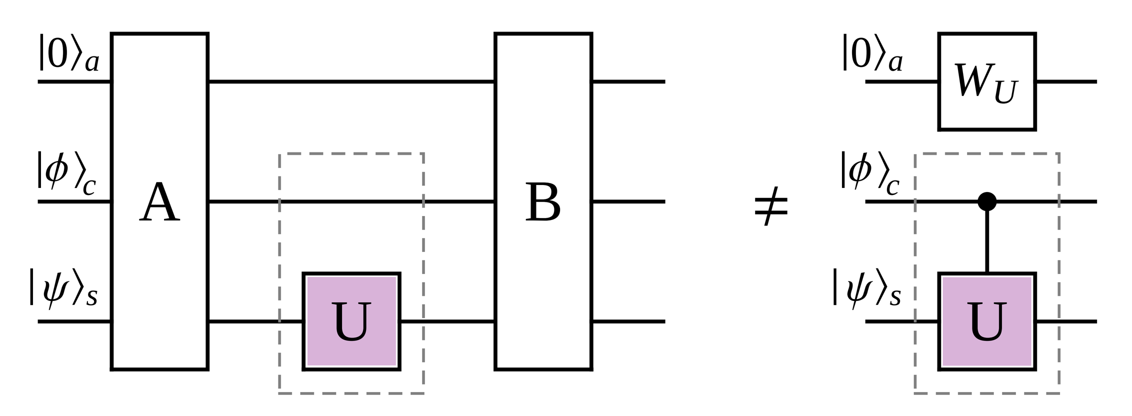
Introduction & Motivation

Recently, there has been renewed interest in the problem of controlling unknown operations [1, 2], previously addressed in an experiment [3], and the related problem of controlling their order [4], within the circuit model of quantum computation. From a computational point of view, it is desirable to equip quantum computers with generic circuits realizing "*if*" clauses that take as their input a number of unknown gates and implement these conditionally on the state of a control qubit. Given an unknown unitary U, one naively expects that it can be inserted into a prefabricated circuit that is independent of U, which performs the operation $\mathrm{ctrl}\text{–}U$. However, various *no-control theorems* [1, 2] show that such constructions are not allowed by the mathematical structure of quantum mechanics. Here [5], we discuss the implications of these theorems, and introduce a novel scheme that allows to add quantum control to unknown unitaries for trapped ions, as well as setups for ions and photons that can realize the quantum-controlled switch of operations.

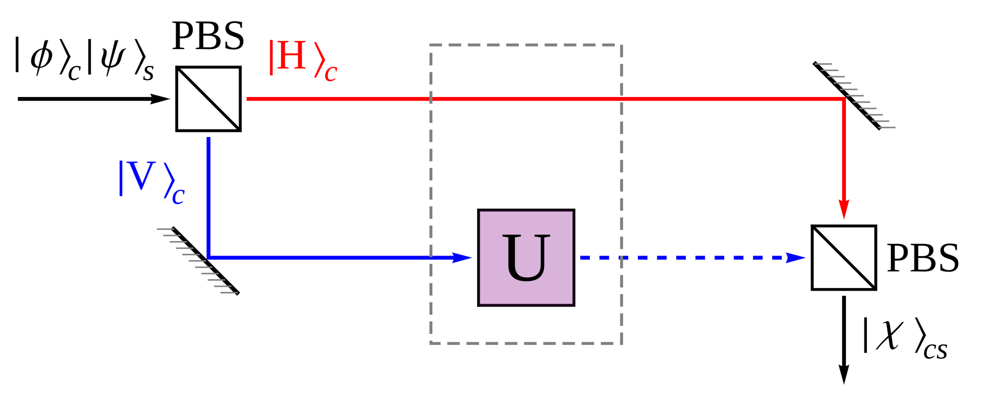
No–Go Theorem (see Refs. [1, 2])



$\nexists$ unitaries A, B such that circuits match for all U

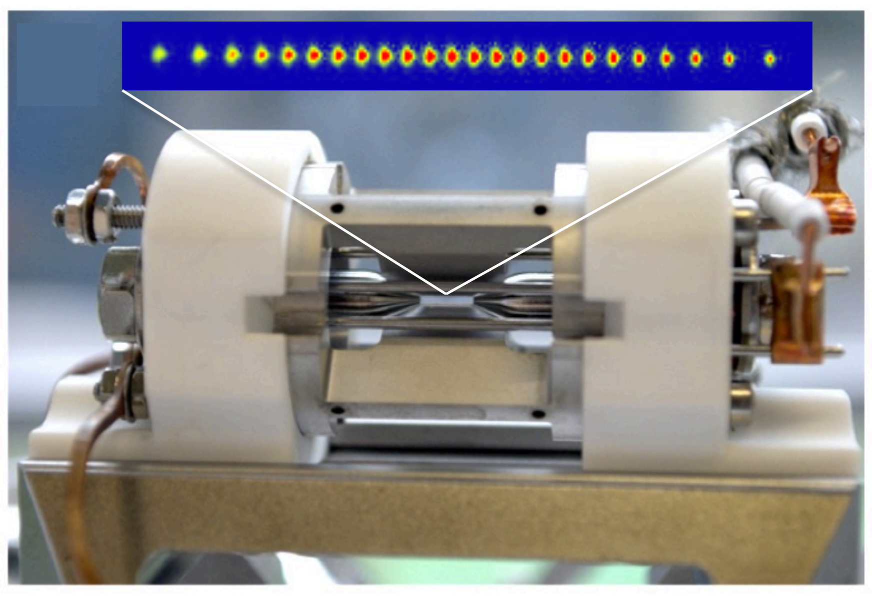
Adding Control Possible in Optical Settings (see Ref. [1, 3]) —Paradox?

- Control in polarization state $|\phi\rangle_c$ of single photon:
  $$|\phi\rangle_c = \alpha\,|\mathrm{H}\rangle_c + \beta\,|\mathrm{V}\rangle_c$$
- U acts on additional degrees of freedom (e.g., orbital angular momentum) encoded in $|\psi\rangle_s$
- Polarizing beam splitters (PBSs) switch control between polarization and spatial degree of freedom
  $$|\chi\rangle_{cs} = \alpha\,|\mathrm{H}\rangle_c|\psi\rangle_s + \beta\,|\mathrm{V}\rangle_c U\,|\psi\rangle_s$$



- How is this possible? Theorem cannot be broken!
- Inspection of Dashed box: visual similarity to circuit, but circuit guarantees single qubit input in each "wire"
- Schematic diagrams, on the other hand, do not generally obey rules of circuit diagram.
- Alternative view: control not "added" to U at all, action of device spatially localized, hence conditioned on position

Ion Traps



Paul trap: individually addressable ions confined in harmonic potential; Common vibrational mode cooled to ground state; Metastable electronic transition encodes qubit; Picture courtesy of B. Lanyon

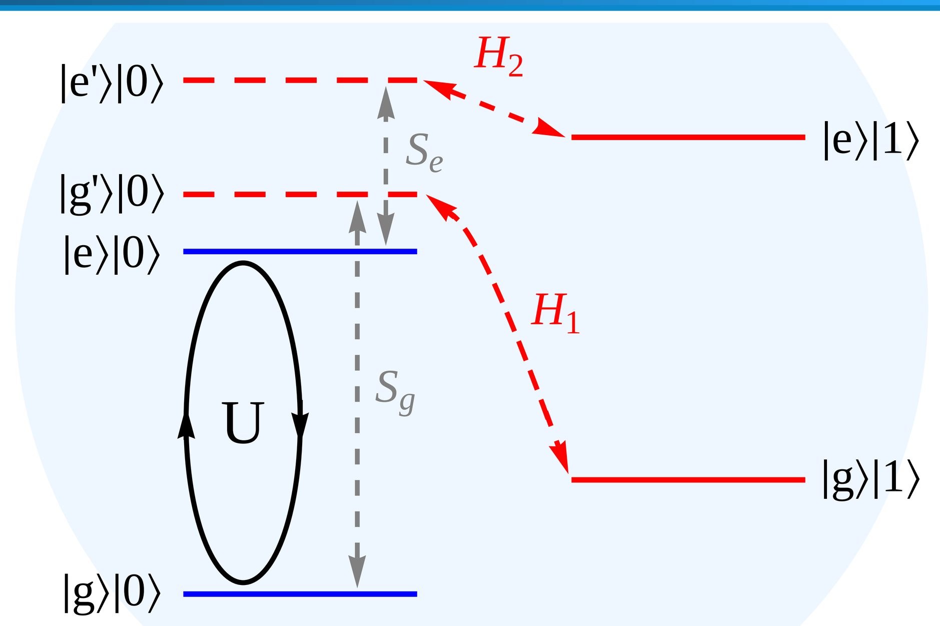
Quantum Control for Trapped Ions (see Ref. [5])

(i) Cirac-Zoller method [6]: blue-detuned laser swaps the qubit state from ion 1 to a common vibrational degree of freedom, driving the transition from $|g\rangle_1|0\rangle$ to $|e\rangle_1|1\rangle$, which transforms the state from $(\alpha\,|g\rangle_1 + \beta\,|e\rangle_1)|\psi\rangle_2|0\rangle$ to $|e\rangle_1|\psi\rangle_2(\alpha\,|1\rangle + \beta\,|0\rangle)$.

(ii) Red-detuned hiding pulses (H_1 and H_2) on ion 2 transfer the populations of $|g\rangle_2|1\rangle$ and $|e\rangle_2|1\rangle$ to the auxiliary levels $|g'\rangle_2|0\rangle$ and $|e'\rangle_2|0\rangle$, respectively, which yields $|e\rangle_1(\alpha\,|\psi'\rangle_2|1\rangle + \beta\,|\psi\rangle_2|0\rangle)$.

(iii) Unknown unitary: The pulse realizing U for ion 2, drives only the transition between $|g\rangle_2|0\rangle$ and $|e\rangle_2|0\rangle$: $|e\rangle_1(\alpha\,|\psi'\rangle_2|1\rangle + \beta\,U\,|\psi\rangle_2|0\rangle)$.

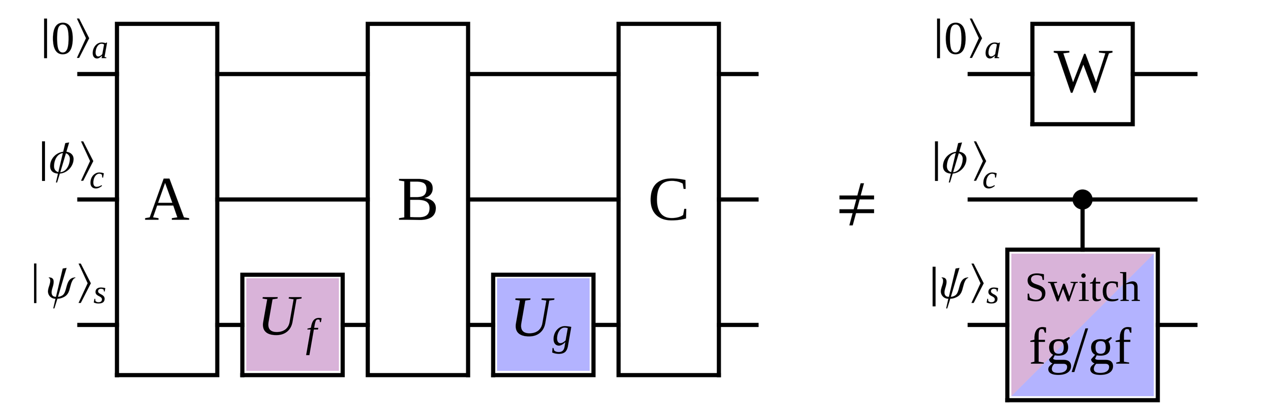
(iv) The hiding pulses H_1 and H_2 are used to reverse the process of step (ii), such that $|\psi'\rangle_2 \to |\psi\rangle_2$.



(v) Final pulse on ion 1 swaps control back from the vibrational mode, $|e\rangle_1|1\rangle \to |g\rangle_1|0\rangle$, which provides $(\alpha\,|g\rangle_1|\psi\rangle_2 + \beta\,|e\rangle_1 U\,|\psi\rangle_2)\,|0\rangle$.

The Controlled Switch (see Ref. [4, 5])

Similar no-go theorem for ordering of unknown operations [4]:

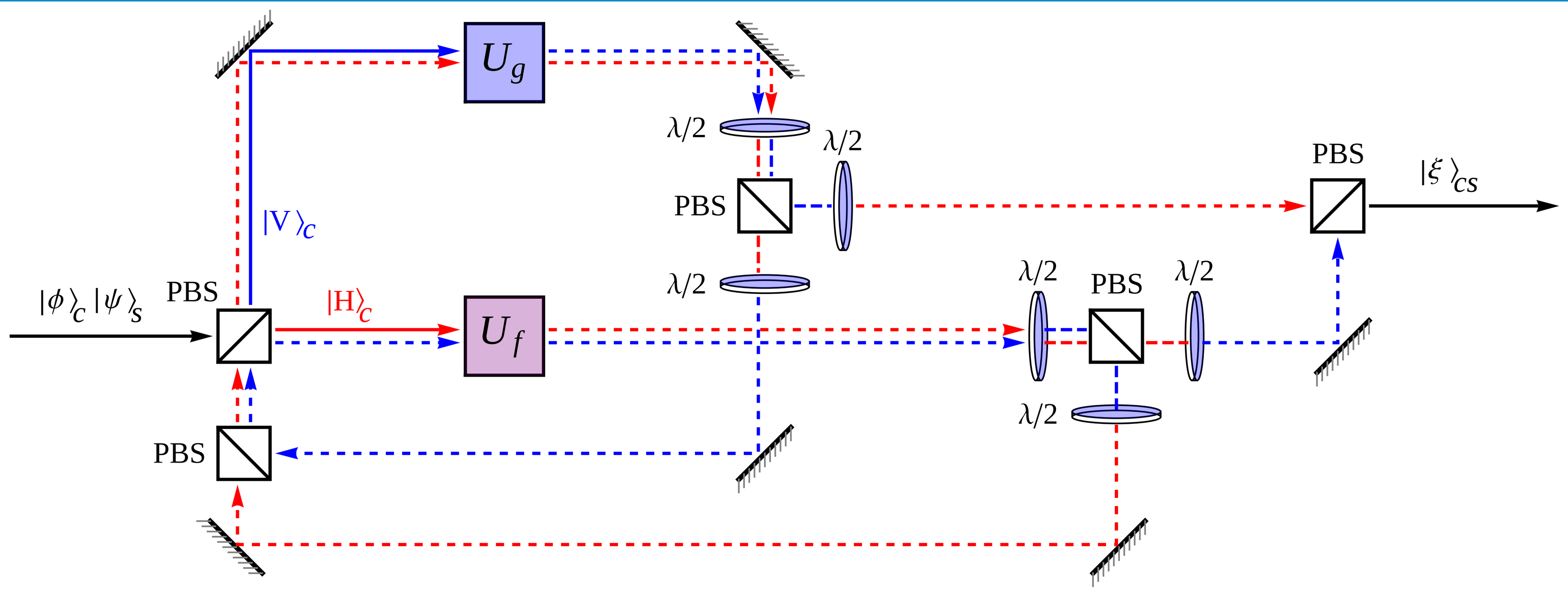


$$\nexists\, A, B, C:\ \forall\, U_f, U_g: C\,(U_g)\,B\,(U_f)\,A \neq W\big(U_gU_f \oplus U_fU_g\big)$$

Ion implementation with "single" laser pulses for U_f and U_g:

- Steps (i),(ii) as before; then (iii) with pulse U_g; reflect used pulse by mirror
- σ_x-like pulses $S_g = |g\rangle\langle g'| + |g'\rangle\langle g|$ and $S_e = |e\rangle\langle e'| + |e'\rangle\langle e|$ (see above)
- Pulse U_f; reflected pulse U_g; repeat S_g, S_e; reflected pulse U_f; (iv),(v) as before

As far as adversary is concerned: pulses used only once, one photon used per pulse



Optical implementation with single devices realizing U_f and U_g:

- Single photon in state $|\phi\rangle_c|\psi\rangle_s$; polarization state $|\phi\rangle_c = \alpha\,|\mathrm{H}\rangle_c + \beta\,|\mathrm{V}\rangle_c$ as control
- State $|\psi\rangle_s$ encoded in additional degree of freedom (e.g., orbital angular momentum)
- Polarizing beam splitters (PBSs) transmit (reflect) horizontal (vertical) polarization
- Half-wave plates ($\lambda/2$) exchange polarizations, while leaving $|\psi\rangle_s$ invariant
- U_f and U_g act only on system; final state $|\xi\rangle_{cs} = \alpha\,|\mathrm{H}\rangle_c\,U_gU_f\,|\psi\rangle_s + \beta\,|\mathrm{V}\rangle_c U_fU_g\,|\psi\rangle_s$

Conclusions

- Have shown: can add control also in ion-trap setup; method easily generalizes to control of n qubits
- Can implement ctrl–Switch in ionic & photonic setups
- No paradoxes; make use of additionally available degrees of freedom or dimensions of physical system
- Other setups: possibility to add control whenever only restricted part of the Hilbert space is used for qubits
- Significance for adversarial settings: provider vs. user

References

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