



Lorentz invariance in quantum information ... a scrutiny

CEQIP 2011
Foundations Session

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- new effects & better understanding of quantum phenomena

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- ▶ study implications for entanglement of two particles and for multipartite entanglement classification
- ▶ and illustrate Lorentz invariance of Bell inequality violation

Relativistic Description

Inertial observers...

related by Poincaré transformations

generally a combination of a spacetime translation a^μ and a Lorentz transformation L^μ_ν

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pure rotations

Inertial observers...

related by **Poincaré transformations**

generally a combination of a spacetime translation a^μ and a Lorentz transformation L^μ_ν

if origin is fixed \Rightarrow **Lorentz transformation L** , combination of

$$R = \begin{pmatrix} 1 & \\ & R(\vec{\varphi}) \end{pmatrix} \quad L_x(\xi) = \begin{pmatrix} \cosh \xi & \sinh \xi & 0 & 0 \\ \sinh \xi & \cosh \xi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

pure rotations & pure boosts (e.g. in x-direction)
with velocity $v = \tanh \xi$

Lorentz group

$$\mathcal{L} = \{ L \in GL(4, \mathbb{R}) \mid L^T \eta L = \eta \}$$

► more precisely: $\mathcal{L}_+^\uparrow = \{ L \in \mathcal{L} \mid \det L = +1, L^0_0 \geq 1 \}$

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succession of two boosts with velocities \vec{v} and \vec{w}

\Rightarrow involves rotation around axis $\vec{v} \times \vec{w}$

Rotation angle

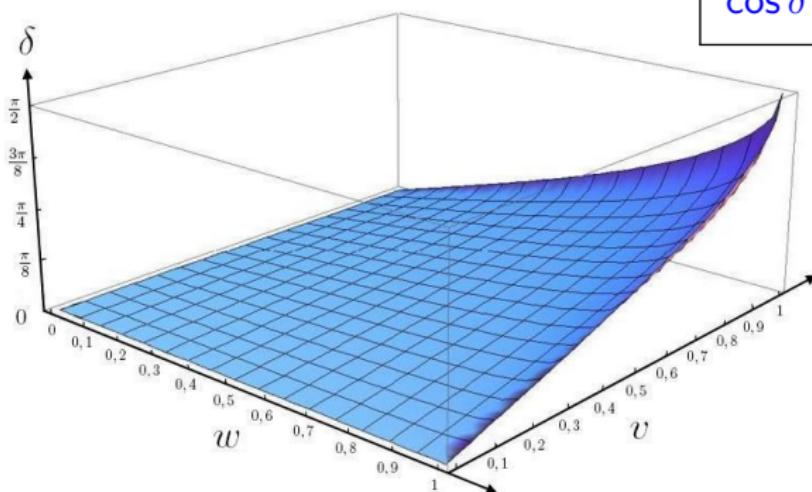


Figure: rotation angle δ displayed as function of boost velocities $v = \frac{|\vec{v}|}{c} = \tanh \xi$, and $w = \frac{|\vec{w}|}{c} = \tanh \chi$, for boosts in perpendicular directions

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then $|p, \sigma\rangle = U(L(p))|k, \sigma\rangle$ “Wigner basis”

Transformation of Single Particle States

Lorentz transformation Λ of momentum eigenstate

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where $W(\Lambda, p) := L^{-1}(\Lambda p) \Lambda L(p)$

Relativistic Entanglement

Consequences of Wigner rotations for single particles

- ▶ uncertainty principle \Rightarrow finite width of momentum space distributions $\Rightarrow | \psi \rangle_{1-particle} = \sum_{\sigma} \int d\mu(p) f_{\sigma}(p) | p, \sigma \rangle$

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- ▶ spin entropy has no invariant meaning ¹

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Consequences of Wigner rotations for entanglement

- ▶ two-particle system: reduced spin density matrix shows
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- ▶ **sum of spin and momentum entanglements** generally **not** invariant ⁴

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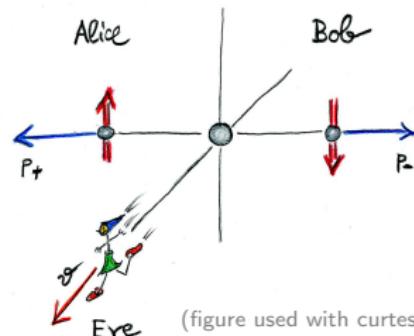
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(figure used with courtesy
of R. Bertlmann)

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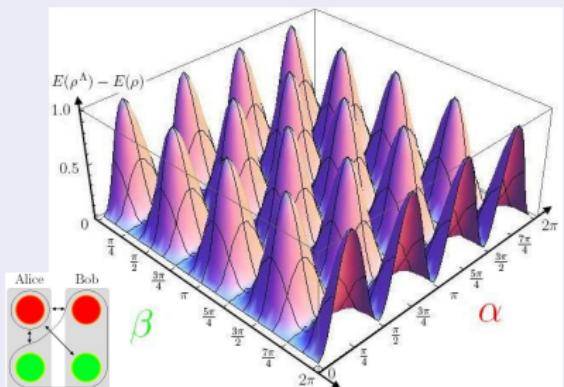
moving in x-direction

\Rightarrow Wigner rotations U_\pm
around $\pm y$ -axis

Entanglement in different Partitions

1 vs 3 Qubit Partition

momentum Λp_+ , Λp_- or spin \uparrow , \downarrow



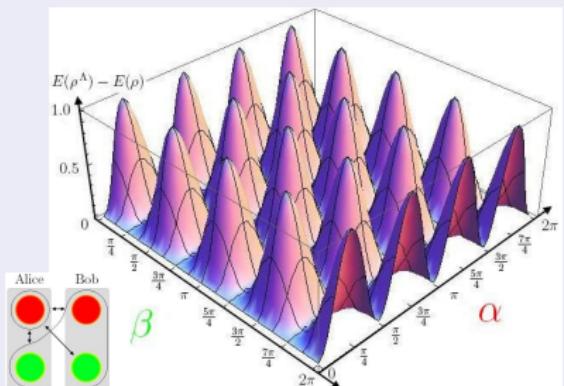
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Difference between linear entropies of initial and
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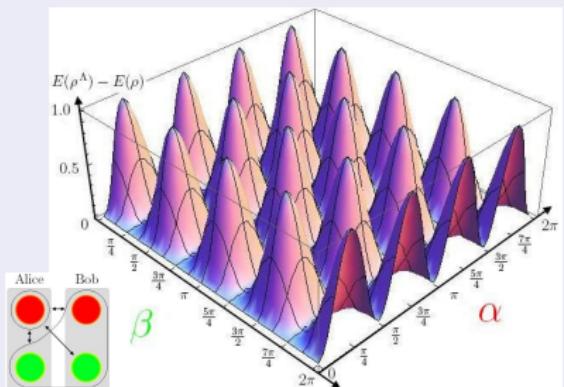
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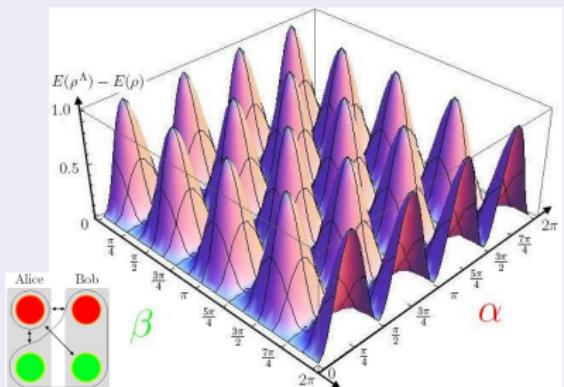
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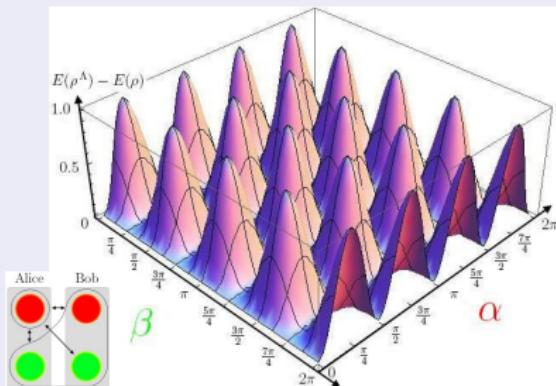
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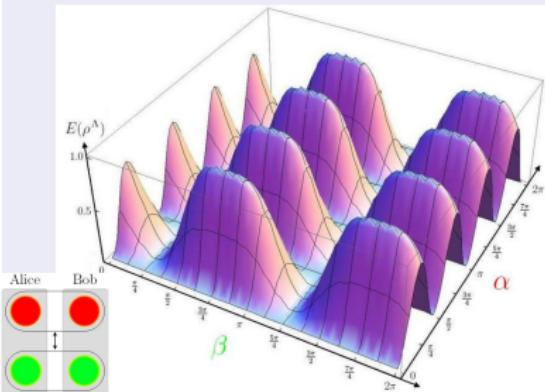
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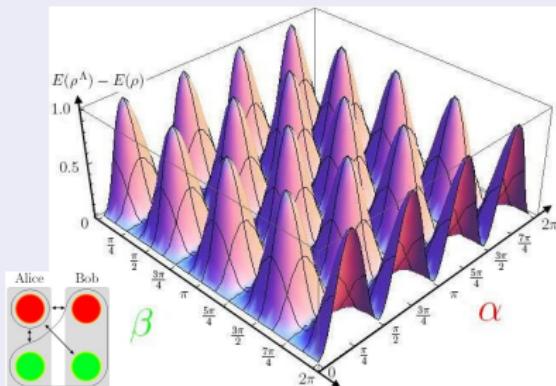


Entanglement change between momentum and spin of $\delta = \pm \frac{\pi}{4}$ Wigner rotated spin-Bell-type state

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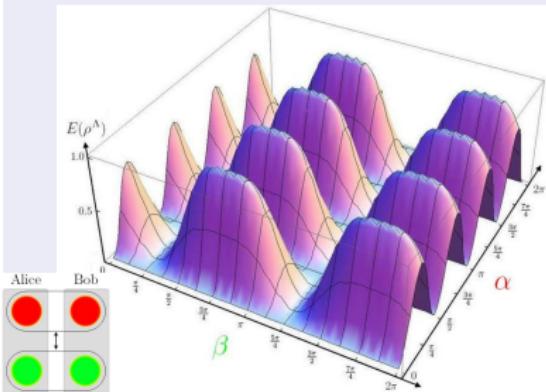
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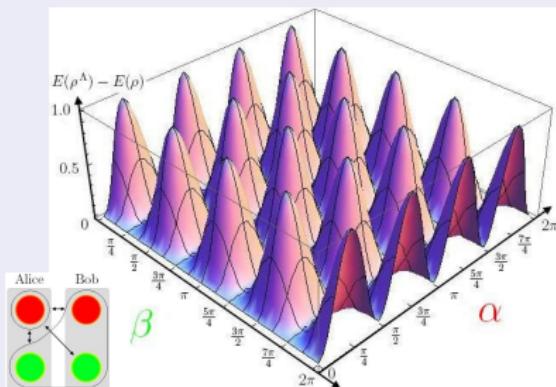
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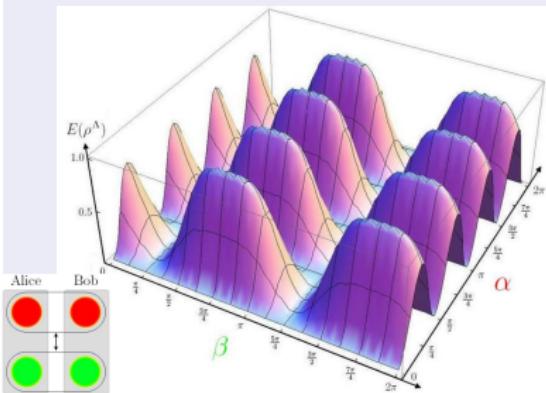
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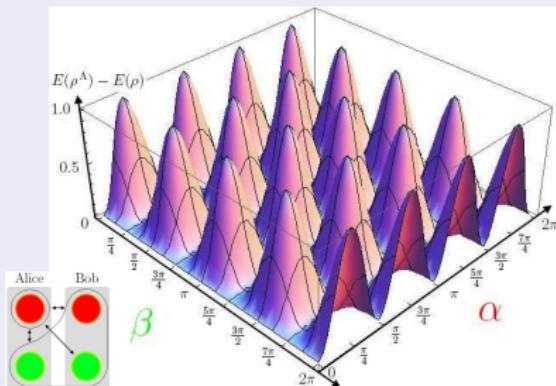
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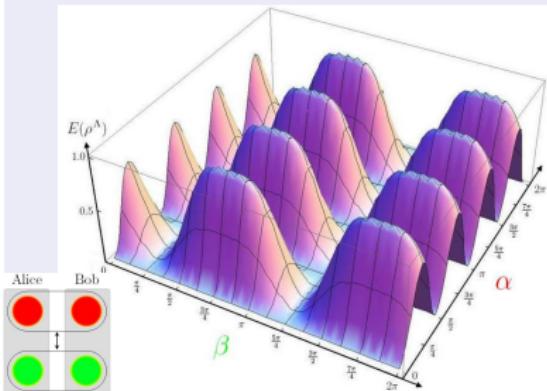
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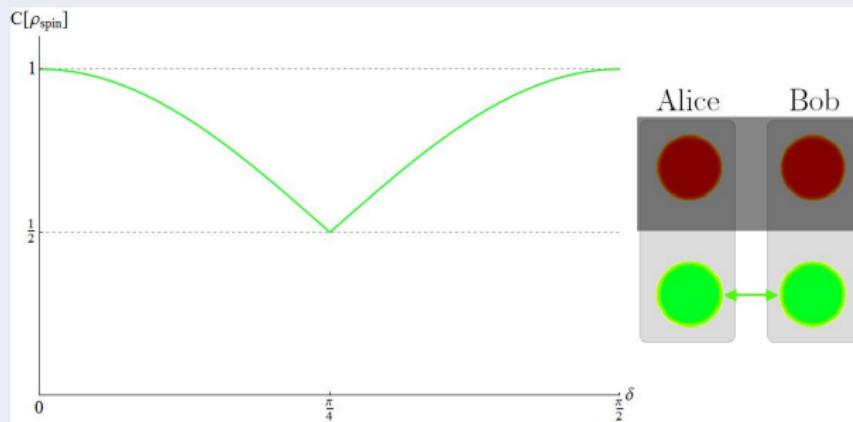
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- ▶ **maximal** change also for entangled spins

Entanglement in different Partitions

Generally (e.g. for ψ^-) there is entanglement - tradeoff between spin - spin entanglement

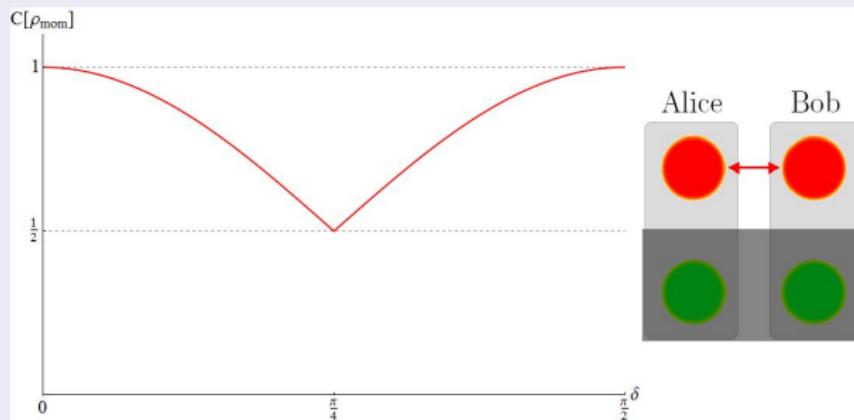
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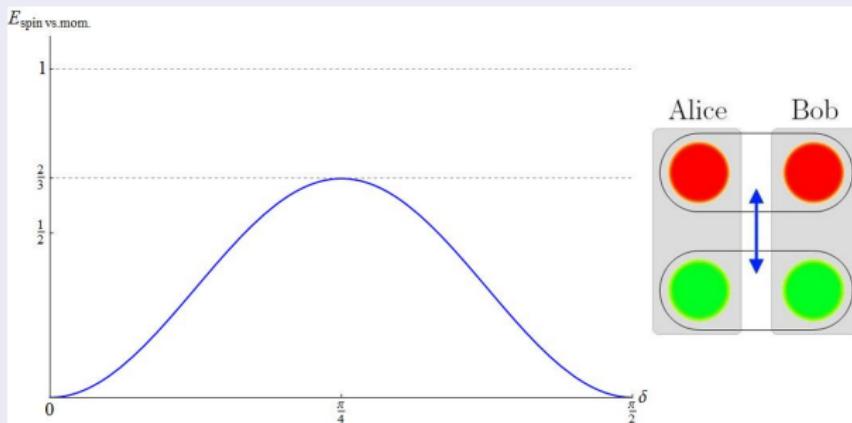
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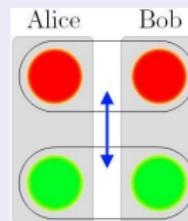
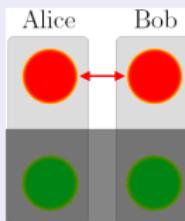
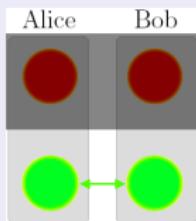
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- ▶ increases for $\delta \rightarrow \frac{\pi}{4}$
- ▶ reduces to zero again in the limit $\delta \rightarrow \frac{\pi}{2}$



Entanglement in different Partitions

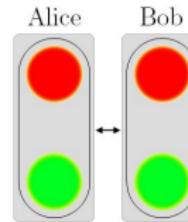
Generally (e.g. for ψ^-) there is entanglement - tradeoff between
spin - spin, momentum - momentum & spin - momentum
entanglement



but entanglement between particles is **genuinely invariant**

Alice - Bob partition

► NO entanglement change



Multipartite Entanglement

Lorentz invariant classification is possible!⁵

Any convex combination of local-unitarily equivalent pure states defines a Lorentz invariant class of genuine multipartite entanglement.

initial state: $\rho = \sum_i q_i |\psi_{mom+spin}^i\rangle\langle\psi_{mom+spin}^i|$

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$$\rho_{spin} = \sum_i q_i \sum_k |\alpha_k^i|^2 |\phi_{spin}^k\rangle_i \langle\phi_{spin}^k|_i$$

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$$\rho_{spin} = \sum_i q_i \sum_k |\alpha_k^i|^2 U(\psi_{mom,i}^k, \Lambda) |\phi_{spin}^k\rangle_i \langle \phi_{spin}^k|_i U^\dagger(\psi_{mom,i}^k, \Lambda)$$

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Bell Inequality Violation

can be maintained in every inertial frame by adjusting measurements and using appropriate observables

Illustration for Pauli - Liubanski Operator:

P-L vector $W^\mu \Rightarrow$ Casimir operator of Poincaré group

spatial part $\vec{W} \Rightarrow$ construct operator $\vec{\sigma}_p = \vec{W}/p^0$

measurement direction: $\vec{a} \implies$ observable: $\hat{a}(p) = \vec{a} \cdot \vec{\sigma}_p$

\Rightarrow relates measurement direction to rest frame direction:

$$\hat{a}(p) = \frac{(L^{-1}(p)a)^i \sigma_i}{|(L^{-1}(p)a)^j|}$$

Bell Inequality Violation

$$\hat{a}(p) = \frac{(L^{-1}(p)a)^i \sigma_i}{|(L^{-1}(p)a)^j|}$$

Observer I

particle momentum: p^μ

measurement direction: a^i

rest frame: \vec{b} : $a^i = (L(p)b)^i$

operator: $\frac{\vec{b}}{|\vec{b}|} \cdot \vec{\sigma}$

Observer II

particle momentum: Λp

measurement direction:
 $a'^i = (\Lambda a)^i$

rest frame: \vec{b} : $a'^i = (\Lambda L(p)b)^i$

operator: $\frac{(L^{-1}(\Lambda p)a')^i \sigma_i}{|(L^{-1}(\Lambda p)a')^j|}$

⇒ Wigner rotations compensated

Possible Bell inequality violation independent of reference frame!

Conclusion

Summary

- ▶ Entanglement not generally invariant

Particle entanglement **is** invariant

Conclusion

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Outlook & Open Questions

- ▶ Which partitions physically accessible?
Separation of **spin** and **momentum** possible?



Conclusion

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- ▶ Entanglement not generally invariant
 - Particle entanglement **is** invariant
- ▶ Multipartite entanglement classification invariant
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Outlook & Open Questions

- ▶ Which partitions physically accessible?
Separation of **spin** and **momentum** possible?
- ▶ Which relativistic observable?



Thank you for your attention