

RELATIVISTIC QUANTUM INFORMATION (RQI)

RQI aims to investigate the relationship of quantum information and relativity:

- Overlap of quantum information, quantum field theory, quantum optics, special- & general relativity
- Identify physical systems to store, process and transmit quantum information in relativistic settings
- Effects of gravity and motion on quantum information: *Unruh-Hawking- & dynamical Casimir effect*

MOTIVATION (SEE REFS. [1, 8])

Experiments are reaching regimes where *relativistic effects* are *non-negligible*, e.g., quantum communication between moving satellites: need toy models & table top setups to inform future space-based experiments.

NON-UNIFORM CAVITY MOTION (SEE REFS. [2, 3, 4, 5])

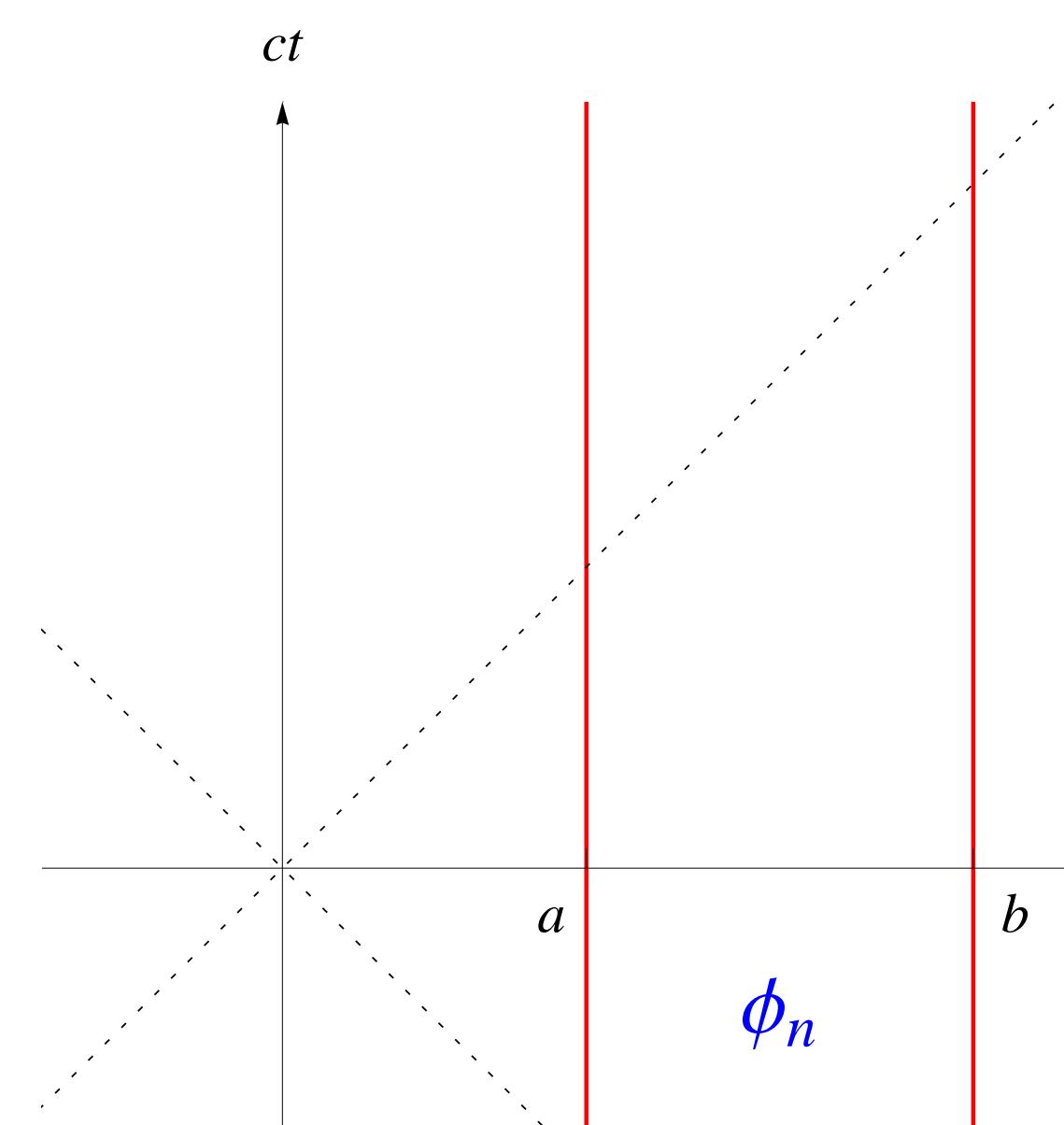


Fig. 1(a): **Inertial cavity** of width $\delta = b - a$

- Real scalar field: $\phi = \sum_n (\phi_n a_n + \phi_n^* a_n^\dagger)$
- Klein-Gordon Equation: $(\partial^\mu \partial_\mu + m^2) \phi(x) = 0$
- Dirichlet boundaries: $\phi(t, \mathbf{a}) = \phi(t, \mathbf{b}) = 0$

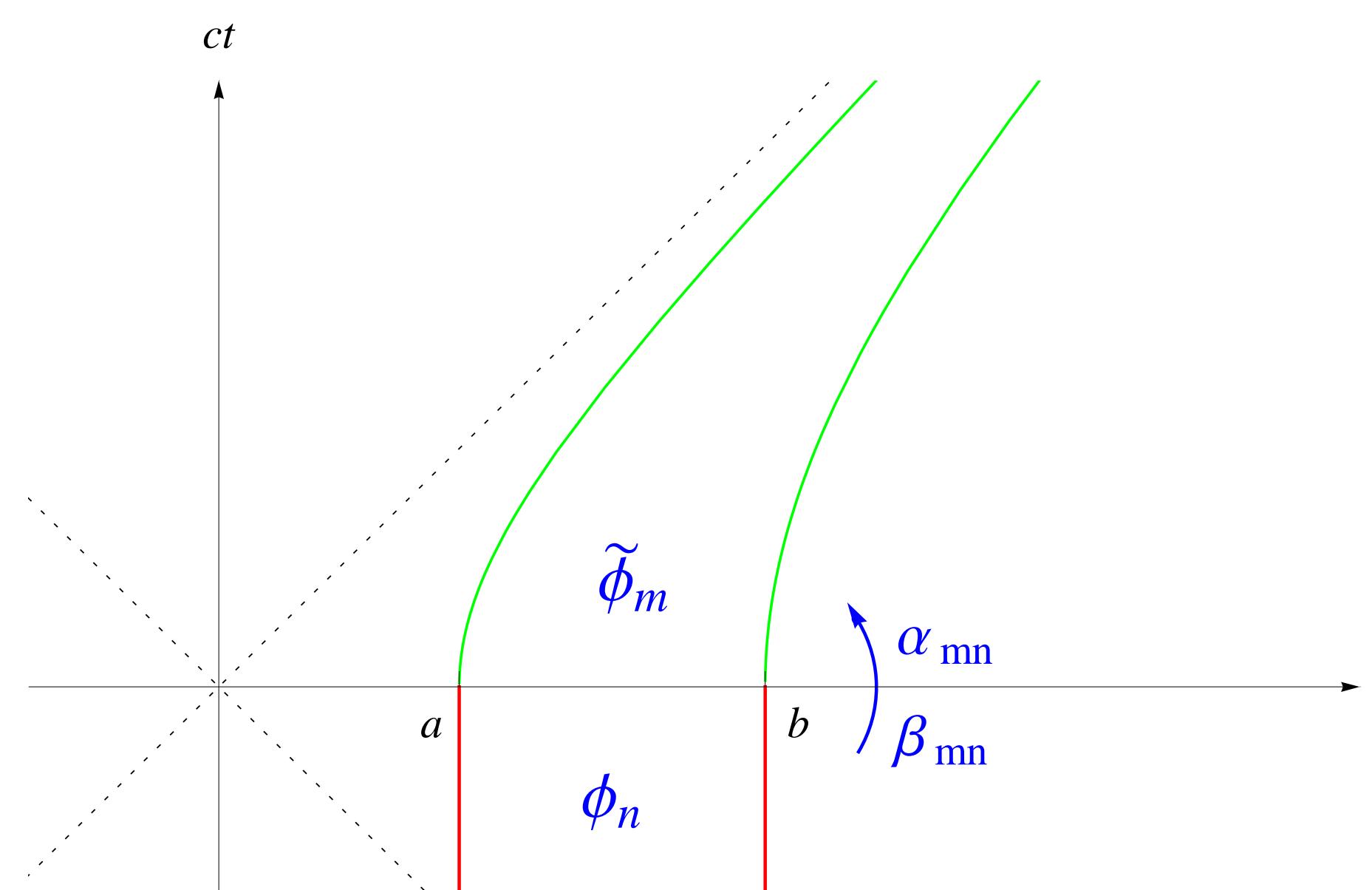


Fig. 1(b): **Sudden acceleration** at $t = 0$, cavity rigid

- Rindler coordinates: $ct = \chi \sinh \eta, x = \chi \cosh \eta$
- Rindler quantization: $\phi = \sum_n (\tilde{\phi}_n \tilde{a}_n + \tilde{\phi}_n^* \tilde{a}_n^\dagger)$
- Bogoliubov transformation: $\phi_n = \sum_m (\alpha_{mn}^* \tilde{\phi}_m - \beta_{mn} \tilde{\phi}_m^*), \alpha_{mn} = (\tilde{\phi}_m, \phi_n), \beta_{mn} = -(\tilde{\phi}_m, \phi_n^*)$

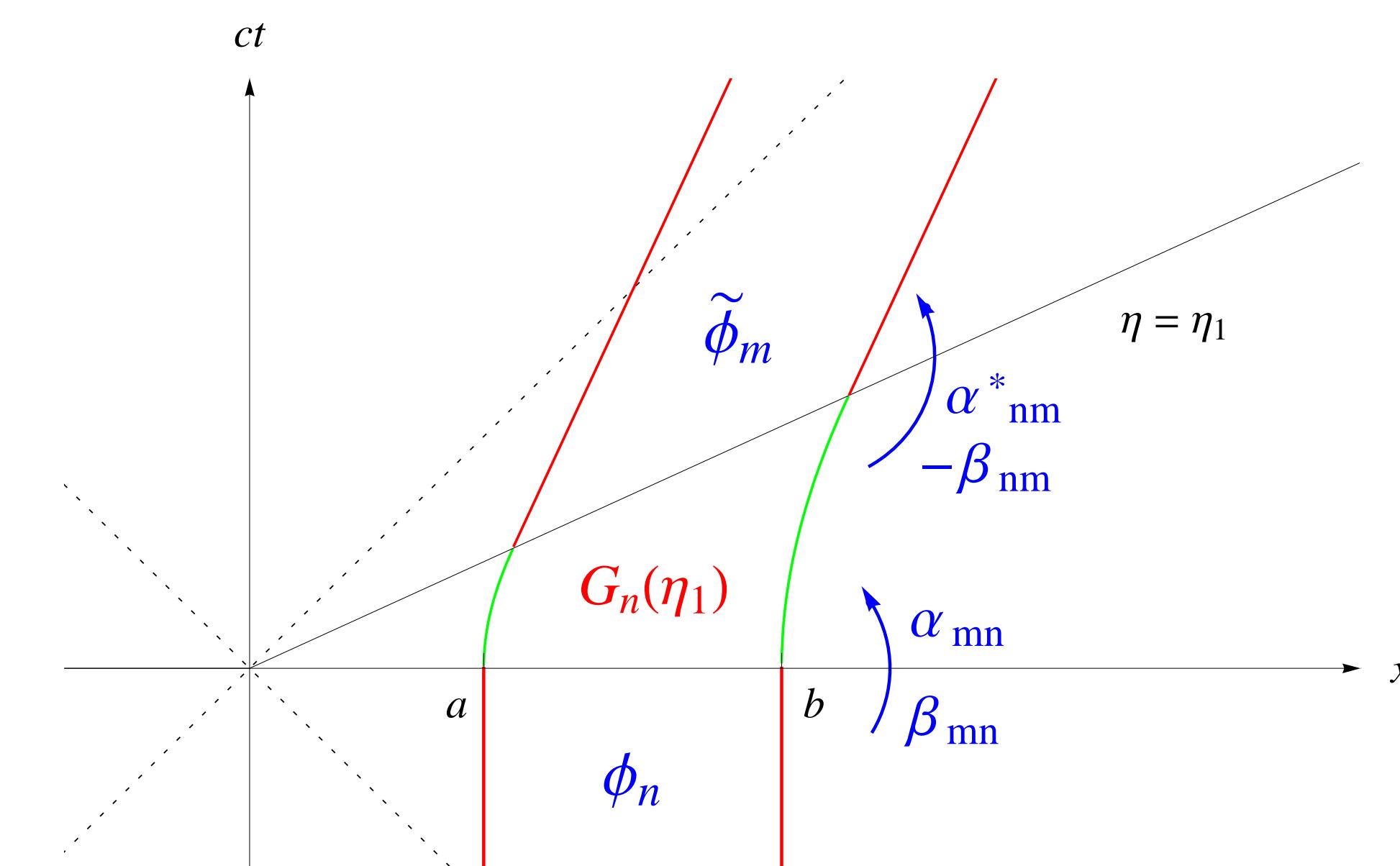


Fig. 1(c): **Finite duration**: modes pick up phases G_n

- Inverse transformation: $\mathcal{A}^{-1}(\alpha, \beta) = \mathcal{A}(\alpha^*, -\beta)$
- Compose transformation $\mathcal{A}^{-1}(\alpha, \beta) G(\eta) \mathcal{A}(\alpha, \beta)$

FOCK STATE PROCEDURE

- Select in-region state: ρ
- Transform to out-region: $\rho \rightarrow \tilde{\rho}$
Vacuum: $|0\rangle = \exp\left(\frac{1}{2} \sum_{p,q} V_{pq} \tilde{a}_p^\dagger \tilde{a}_q^\dagger\right) |\tilde{0}\rangle$
where $V = -\beta^* \alpha^{-1}$
- Fock states: act on $|0\rangle$ with creation operators
 $a_n^\dagger = \sum_m (\alpha_{mn}^* \tilde{a}_m^\dagger + \beta_{mn} \tilde{a}_m)$
- Trace out inaccessible modes
- Study entanglement properties

GAUSSIAN STATES — CONTINUOUS VARIABLES (CV) (SEE REF. [6])

Covariance matrix: $\Gamma_{ij} = \langle \mathbb{X}_i \mathbb{X}_j + \mathbb{X}_j \mathbb{X}_i \rangle_\rho - 2 \langle \mathbb{X}_i \rangle_\rho \langle \mathbb{X}_j \rangle_\rho$

Quadratures: $\mathbb{X}_{(2n-1)} = \frac{1}{\sqrt{2}} (a_n + a_n^\dagger), \mathbb{X}_{(2n)} = \frac{-i}{\sqrt{2}} (a_n - a_n^\dagger)$

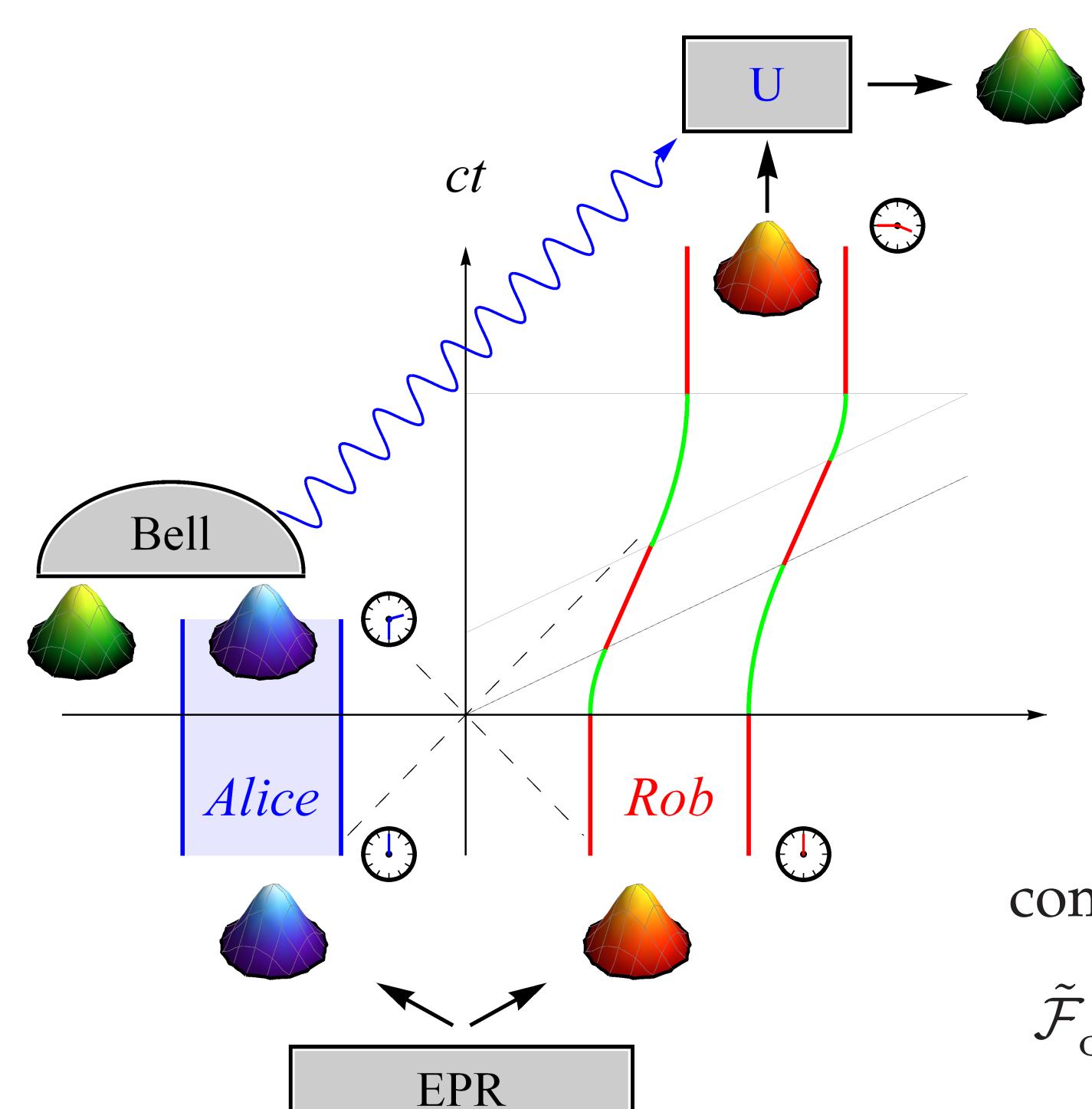
Bogoliubov transformation \rightarrow symplectic transformation S

$$\Gamma = \begin{pmatrix} C_{11} & C_{12} & C_{13} & \dots \\ C_{21} & C_{22} & C_{23} & \dots \\ C_{31} & C_{32} & C_{33} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \rightarrow \tilde{\Gamma} = \begin{pmatrix} \tilde{C}_{11} & \tilde{C}_{12} & \tilde{C}_{13} & \dots \\ \tilde{C}_{21} & \tilde{C}_{22} & \tilde{C}_{23} & \dots \\ \tilde{C}_{31} & \tilde{C}_{32} & \tilde{C}_{33} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad \mathcal{M}_{mn} = \text{Re} \begin{pmatrix} \alpha_{mn} - \beta_{mn} & -i(\alpha_{mn} + \beta_{mn}) \\ i(\alpha_{mn} - \beta_{mn}) & \alpha_{mn} + \beta_{mn} \end{pmatrix}$$

where $\tilde{C}_{mn} = \sum_{i,j} \mathcal{M}_{mi} C_{ij} \mathcal{M}_{nj}^T$

Trace out/remove inaccessible modes \Rightarrow study entanglement between remaining modes

TELEPORTATION IN MOTION (SEE REF. [7])



Alice wants to teleport a coherent state — standard CV protocol:

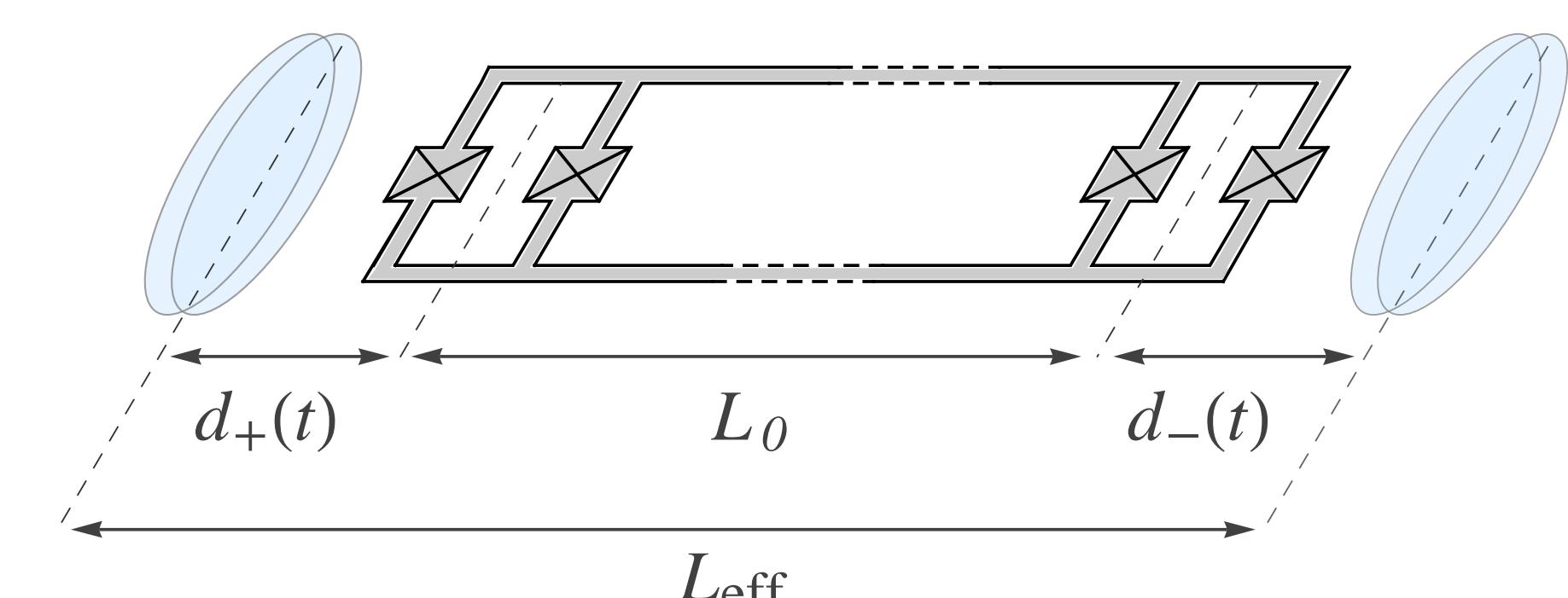
- Resource: two-mode squeezing r : modes k (Alice) & k' (Rob)
- Alice's Bell measurement: double homodyne detection
- Results: sent to Rob classically to retrieve teleported state

Fidelity of teleportation *degraded by Rob's motion*:

Coefficients: $\alpha_{mn} = \alpha_{mn}^{(0)} + \alpha_{mn}^{(1)} h + O(h^2), \beta_{mn} = \beta_{mn}^{(1)} h + O(h^2)$
 $h := \frac{\delta \dot{v}}{c^2},$ with \dot{v} ... acceleration at center of cavity

Fidelity optimized over local Gaussian operations: $\tilde{\mathcal{F}}_{\text{opt}}$ compensates phases from time evolution $\Rightarrow \tilde{\mathcal{F}}_{\text{opt}} = \tilde{\mathcal{F}}_{\text{opt}}^{(0)} - \tilde{\mathcal{F}}_{\text{opt}}^{(2)} h^2 + \mathcal{O}(h^4)$
 $\tilde{\mathcal{F}}_{\text{opt}}^{(0)} = (1 + e^{-2r})^{-1}, \tilde{\mathcal{F}}_{\text{opt}}^{(2)} = \tilde{\mathcal{F}}_{\text{opt}}^{(0)} \frac{1}{2} \sum_{n \neq k'} (|\beta_{nk'}^{(1)}|^2 + |\alpha_{nk'}^{(1)}|^2 \tanh(2r))$

SIMULATION (SEE REF. [7, 8])



1-dim transmission line for microwave radiation terminated by superconducting circuits (SQUIDs)
 \Rightarrow simulate boundary conditions for a cavity of effective length $L_{\text{eff}} = L_0 + d_+(t) + d_-(t)$.
Predicted degradation of optimal teleportation fidelity for realistic parameters: $\approx 4\%$ of $\tilde{\mathcal{F}}_{\text{opt}}^{(0)}$.

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