Thermodynamics and Energetics of Creating Correlations

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Introduction & Motivation

A fundamental connection between thermodynamics and information theory arises from the fact that correlations exhibit an inherent work value [1]. Conversely, one may ask how much energy is needed to create correlations [2, 3]. Here, we present investigations of this question for both noninteracting systems [3], and for systems where interactions cannot be controlled or removed [4]. We present ultimate bounds and optimal protocols for the noninteracting case, and discuss general strategies outperforming those bounds for naturally coupled systems. Our results are illustrated for some examples of interest.

General Setup

Systems S_1 and S_2 at temperature T

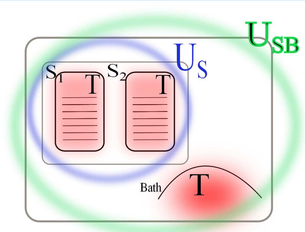
Extracted from thermal bath B at T

Thermal state: $\tau(\beta) = \mathcal{Z}^{-1} e^{-\beta H}$

U_S & U_{SB} create correlations

Work cost: $W = \Delta F$

Free energy: $F(\rho) = E(\rho) - TS(\rho)$



Optimal Generation of Correlations (see Ref. [3])

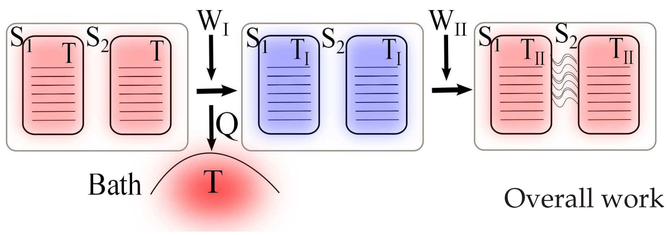
Bound for maximal correlation for $H_{SB} = H_{S_1} + H_{S_2} + H_B$

Mutual information: $\mathcal{I}_{S_1 S_2}(\rho_S) = S(\rho_{S_1}) + S(\rho_{S_2}) - S(\rho_S)$

$W = \Delta F_S + \Delta F_B + T\mathcal{I}_{SB}$ & $\Delta F_S = \Delta F_{S_1} + \Delta F_{S_2} + T\mathcal{I}_{S_1 S_2}$

For thermal states: $\Delta F = T\, S(\rho \| \tau(\beta))$

Maximal correlation for fixed W and T: $\mathcal{I}_{S_1 S_2} \leq \beta W$



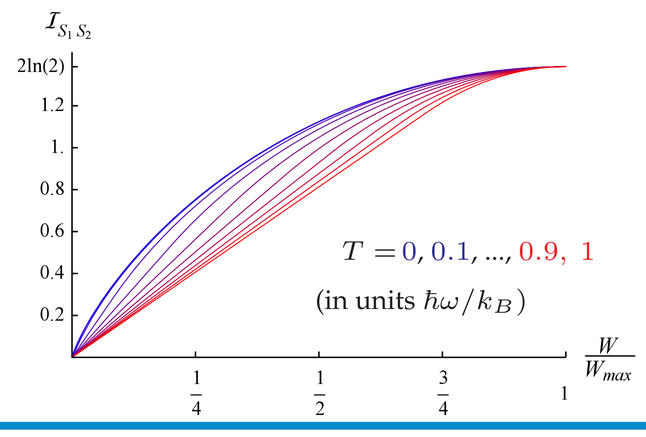
Optimal protocol

I Cooling: Lower temperature of S from T to $T_I \leq T$

II Correlating: Isolate S from B; Correlate via unitary U_S such that S_1 and S_2 are locally thermal at temperature $T_{II} \geq T_I$

Overall work cost: $W = W_I + W_{II} = TS(\tau_S(\beta_{II}) \| \tau_S(\beta)) + T\mathcal{I}_{S_1 S_2} \Rightarrow T_{II} = T$

Example: Correlations for Two Fermionic Modes



Optimal protocol distinguishes **two regimes**:

Low energy: $\mathcal{I}_{S_1 S_2} = \beta W$ if $W \leq T\, S(\tau_S(\beta))$

High energy: $\mathcal{I}_{S_1 S_2} = S(\tau_S(\beta_{II}))$ if $W > T\, S(\tau_S(\beta))$

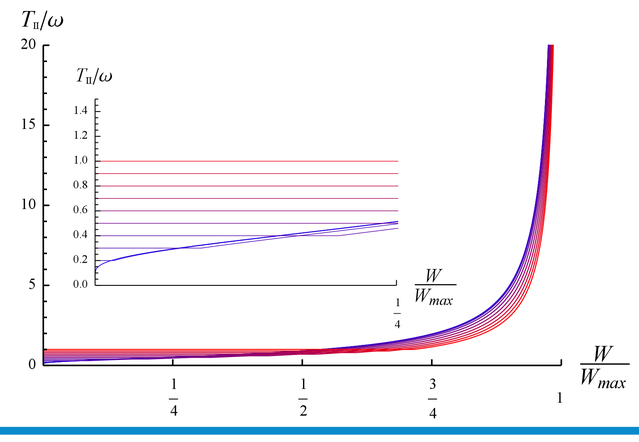
Example: **two fermionic modes** (not qubits! [5]) with frequency ω

Fermi-Dirac statistics: partition function $\mathcal{Z}_{FD} = 1 + e^{-\beta}$

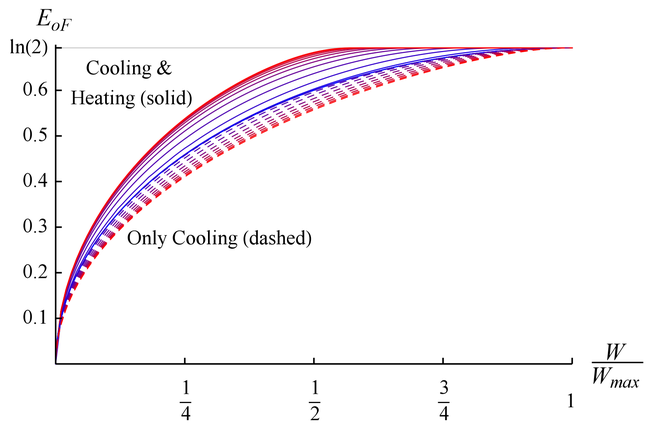
Thermal state: average particle number $N_{S_1} = N_{S_2} = (e^{\beta} + 1)^{-1}$

Pauli principle: $0 \leq N_{S_i} \leq 1$ for any state

$\Rightarrow$ maximally useful energy for protocol: $W_{max} = 2T \ln(e^{\beta} + 1) - \omega$



Generation of Entanglement for Two Fermionic Modes



Energy cost of entanglement: in general difficult to answer & depends on the choice of measure

Entanglement of Formation:

$$E_{oF} = \inf_{\mathcal{D}(\rho_S)} \sum_i p_i\, \mathcal{E}(|\psi_i\rangle)$$

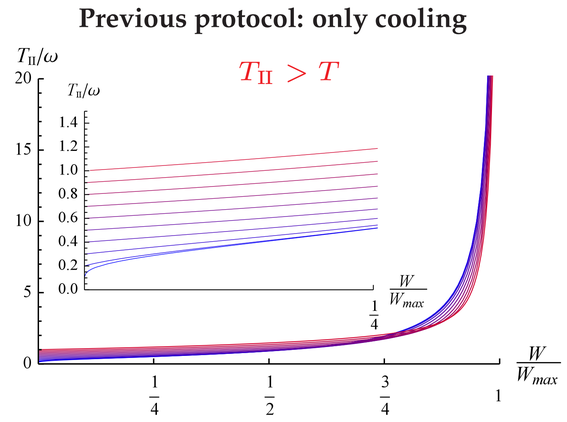
where $\mathcal{E}(|\psi_i\rangle) = S\left(\mathrm{Tr}_{S_2}(|\psi_i\rangle\langle\psi_i|)\right)$

$\mathcal{D}(\rho_S) = \{p_i, |\psi_i\rangle \mid \sum_i p_i |\psi_i\rangle\langle\psi_i| = \rho_S\}$

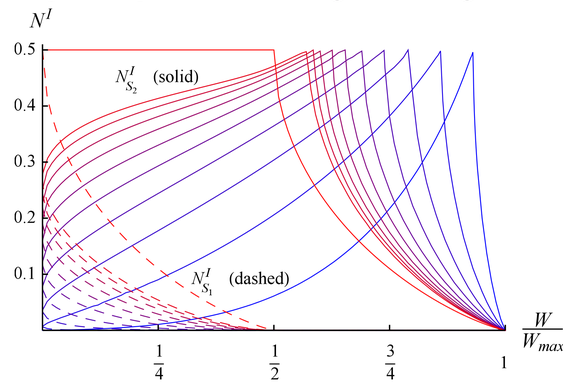
Fermions: parity superselection rule [6]

$\Rightarrow$ allowed state space [and $\mathcal{D}(\rho_S)$] restricted

Previous protocol: only cooling



Optimal protocol: cooling & heating



Role of Interactions [4]

Interacting systems: $H_S = H_{S_1} + H_{S_2} + H_I$

typically initially correlated at any temperature

Example: Fermions with $H_{S_1} + H_{S_2} = \omega(b_1^\dagger b_1 + b_2^\dagger b_2)$

with $H_I = \epsilon_{\text{even}}(b_1 b_2 + b_2^\dagger b_1^\dagger) + \epsilon_{\text{odd}}(b_1^\dagger b_2 + b_2^\dagger b_1)$

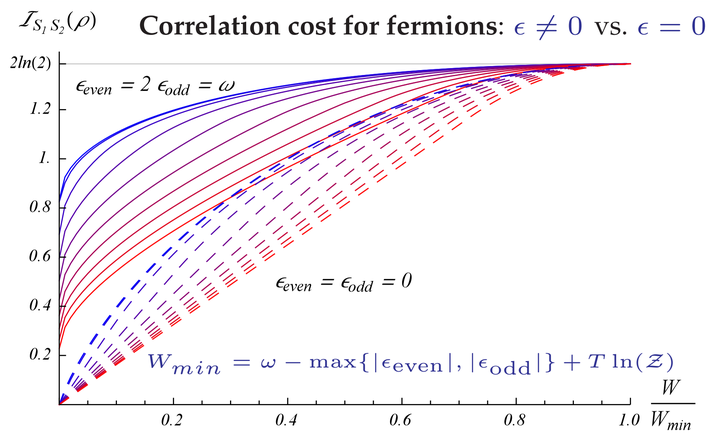
Spectrum of H_S: $\lambda_{1,4} = \omega \pm \sqrt{\omega^2 + \epsilon_{\text{even}}^2}$

$\| \lambda_{1,4} \rangle\rangle = \frac{1}{\sqrt{\epsilon_{\text{even}}^2 + \lambda_{1,4}^2}} \left(\epsilon_{\text{even}} \| 0 \rangle\rangle - \lambda_{1,4} \| 1_1 \rangle\rangle \| 1_2 \rangle\rangle \right)$

$\lambda_{2,3} = \omega \pm \epsilon_{\text{odd}}$ & $\| \lambda_{2,3} \rangle\rangle = \frac{1}{\sqrt{2}} \left(\| 1_2 \rangle\rangle \pm \| 1_1 \rangle\rangle \right)$

Initial correlation depends on $\epsilon_{\text{even}}/\omega$, $\epsilon_{\text{odd}}/\omega$ & T

Correlation cost for fermions: $\epsilon \neq 0$ vs. $\epsilon = 0$



General relation between work and correlation gain: $\tau \to \rho$

$\Delta F_S = T\Delta\mathcal{I}_{S_1 S_2} + \mathrm{Tr}(H_I[\rho - \tau]) + \Delta\tilde{F}_{S_1} + \Delta\tilde{F}_{S_2}$

Nonequilibrium free energies w.r.t. the local Hamiltonians

$\tilde{F}_{S_i}(\rho) = \mathrm{Tr}(H_{S_i}\rho) - TS(\rho_{S_i})$

Initial marginals $\tau_{S_i} \neq$ local Gibbs states $\gamma_{S_i} \equiv \mathcal{Z}_{S_i}^{-1} e^{-\beta H_{S_i}}$

General strategies to lower correlation cost

(1) Fix $\mathrm{Tr}(H_I[\rho - \tau])$ & lower $\Delta\tilde{F}_{S_i}$: move $\rho_{S_i} \to \gamma_{S_i}$

(2) Fix $\Delta\tilde{F}_{S_i}$ & "anti-align" correlation and interaction vectors

Strategies limited by positivity constraints

Conclusions

- Correlations have intrinsic work cost!
- Noninteracting systems: $\mathcal{I}_{S_1 S_2} \leq \beta W$
- Optimal protocol attains this bound (small W regime)
- Interactions: correlations can be larger $\mathcal{I}_{S_1 S_2} > \beta W$
- General strategies to achieve larger overall correlations
- But: new correlations $\Delta\mathcal{I}_{S_1 S_2}$ can be more expensive
- Examples for qubits, bosonic and fermionic modes

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