The Thermodynamics of Creating Correlations

Limitations and Optimal Protocols

Group Retreat 2014

Maria Waldrast, September 28th - October 1st, 2014

Nicolai Friis

Institute for Quantum Optics and Quantum Information - Innsbruck

work in collaboration with *David E. Bruschi* (Jerusalem), *Martí Perarnau-Llobet*, *Karen V. Hovhannisyan*, and *Marcus Huber* (Barcelona)

Motivation — Quantum Thermodynamics

Some approaches to thermodynamics in the quantum domain

- Landauer's principle: minimum energy for bit erasure (see, e.g., Reeb & Wolf, arXiv:1306.4352; Esposito & Van den Broeck, Europhys. Lett. **95**, 40004 (2011) [arXiv:1104.5165])
- Role of quantum effects in thermal machines

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  Correa *et al.*, Phys. Rev. E **87**, 042131 (2013) [arXiv:1212.4501];
  Brunner *et al.*, Phys. Rev. E **89**, 032115 (2014) [arXiv:1305.6009];
  Gallego, Riera, & Eisert, arXiv:1310.8349;
- Thermodynamic laws in quantum regime (see, e.g., Brandão *et al.*, arXiv:1305.5278)

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thermodynamics	$\longleftrightarrow$	quantum information
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Outline

We ask[1]

How strongly can one correlate a system at initial temperature T if the work W is supplied?

Previous work in this direction

- All correlations imply extractable work (Perarnau-Llobet *et al.*,arXiv:1407.7765)
- Cost of correlating isolated systems of qubits (Huber *et al.*, arXiv:1404.2169)

Outline of our contribution[1]

Generating total correlations:

- Bound for overall correlations measured by the mutual information
- Optimal protocol saturating this bound

Generating entanglement — bottom-up approach:

- Optimal entanglement generation between two bosonic modes
- Optimal entanglement generation between two fermionic modes

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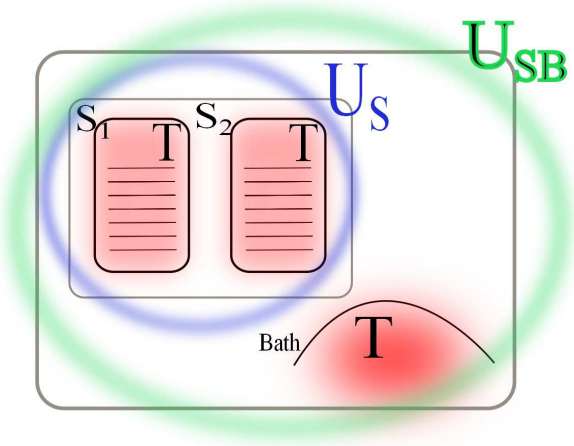
Framework



System S & Bath B: $\mathcal{H}_{S_1} \otimes \mathcal{H}_{S_2} \otimes \mathcal{H}_B$

non-interacting $H_{SB} = H_{S_1} + H_{S_2} + H_B$

thermal state $\tau_{SB}(\beta) = \mathcal{Z}^{-1}(\beta)\, e^{-\beta H_{SB}}$

initial state uncorrelated $\tau_{SB} = \tau_{S_1} \otimes \tau_{S_2} \otimes \tau_B$

Cost of moving the system out of thermal equilibrium

Free energy difference: $\Delta F(\tau(\beta) \to \rho) = F(\rho) - F(\tau(\beta)) \geq 0$

where $F(\rho) = \mathcal{E}(\rho) - T\mathcal{S}(\rho)$

energy entropy

Applying unitaries (U_S or U_{SB}) to correlate system $\Leftrightarrow$ energy cost

Notation: $\mathcal{Z}$ ... partition function, $\beta = 1/T$, units: $\hbar = k_B = 1$

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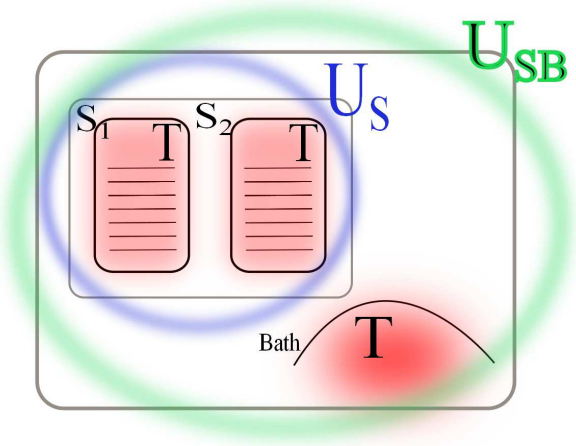

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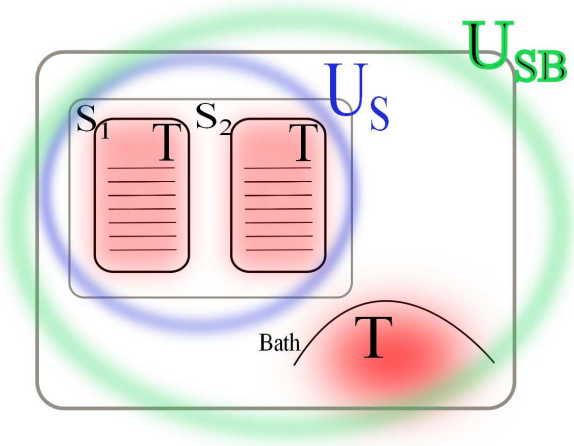

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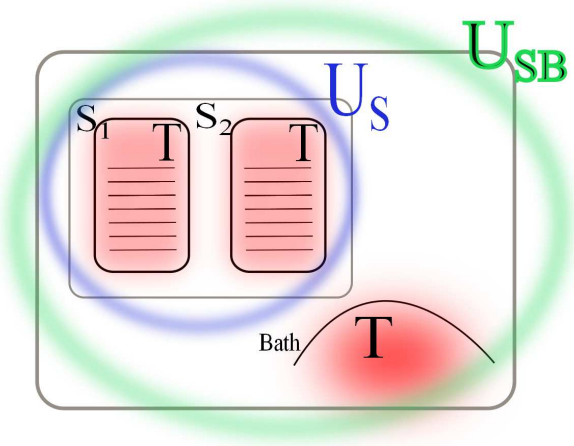

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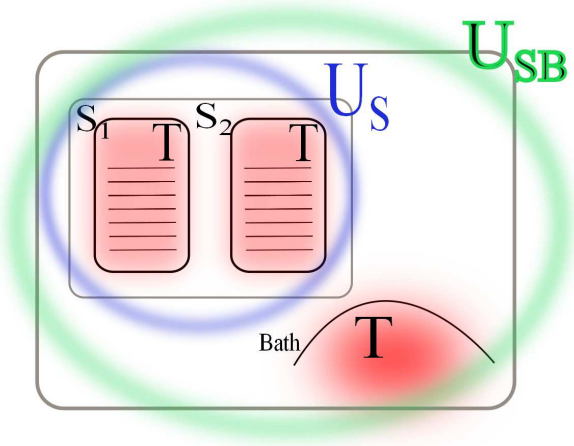

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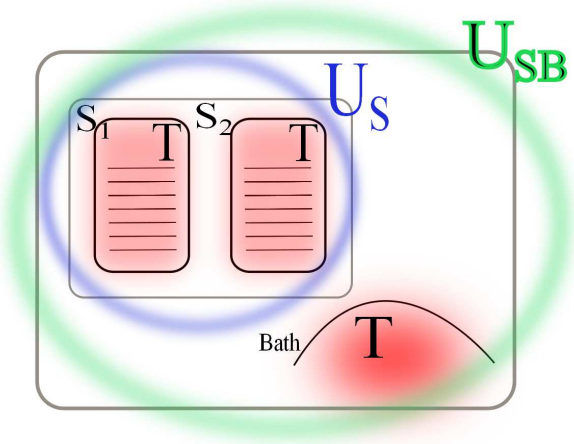

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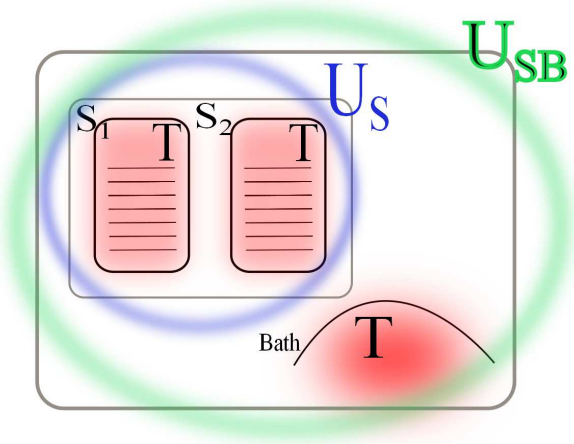

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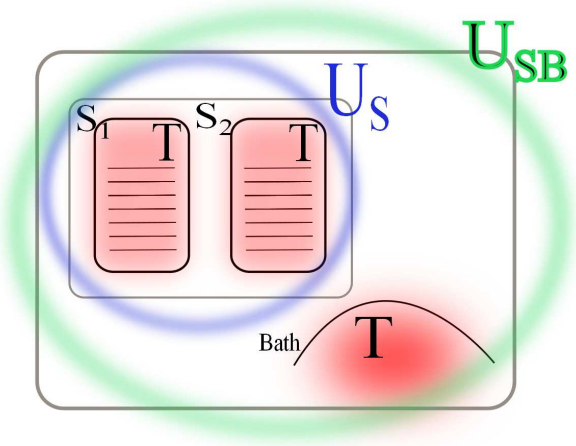

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Optimal creation of total correlations

Bound on optimal correlations

Measure of correlation

Mutual information: $\mathcal{I}_{S_1 S_2}(\rho_S) = S(\rho_{S_1}) + S(\rho_{S_2}) - S(\rho_S)$

Energy cost for global unitary U_{SB}

$$W = \Delta E_S + \Delta E_B$$

$$= \Delta S(\rho_S) + \Delta S(\rho_B) - \Delta S(\rho_{SB})$$

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Maximal correlation for fixed temperature and work

$$\mathcal{I}_{S_1 S_2} \leq \beta W$$

For arbitrarily large bath $\Delta F_B = 0$, $\mathcal{I}_{SB} = 0$ achievable

(see Åberg, Nat. Commun. **4**, 1925 (2013) [arXiv:1110.6121])

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Maximal correlation for fixed temperature and work

$$\mathcal{I}_{S_1 S_2} \leq \beta W$$

For arbitrarily large bath $\Delta F_B = 0$, $\mathcal{I}_{SB} = 0$ achievable

(see Åberg, Nat. Commun. **4**, 1925 (2013) [arXiv:1110.6121])

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Optimal protocol for the generation of mutual information

Without loss of generality we may split the protocol into two steps

(I) **Cooling**: Lower temperature of S from T to $T_{\mathrm{I}} \leq T$

Minimal energy cost $W_{\mathrm{I}} = \Delta F_S = F(\tau_S(\beta_{\mathrm{I}})) - F(\tau_S(\beta))$

(II) **Correlating**: Isolate system from bath

Correlate via unitary U_S such that S_1 and S_2 are locally thermal at temperature $T_{\mathrm{II}} \geq T_{\mathrm{I}}$,

$\Rightarrow$ ensures correlation at minimal energy cost (see Aubrey et al. arXiv:1104.2018)

Energy cost $W_{\mathrm{II}} = D(\tau_S(\beta_{\mathrm{II}})) - F(\tau_S(\beta_{\mathrm{I}}))$

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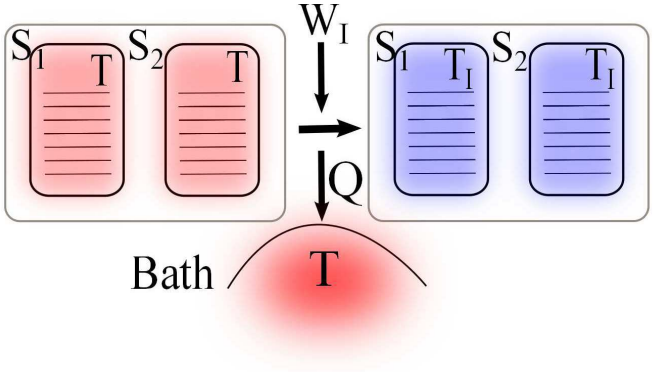

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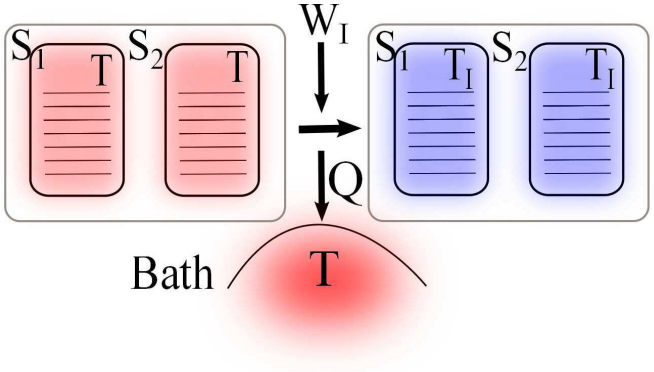

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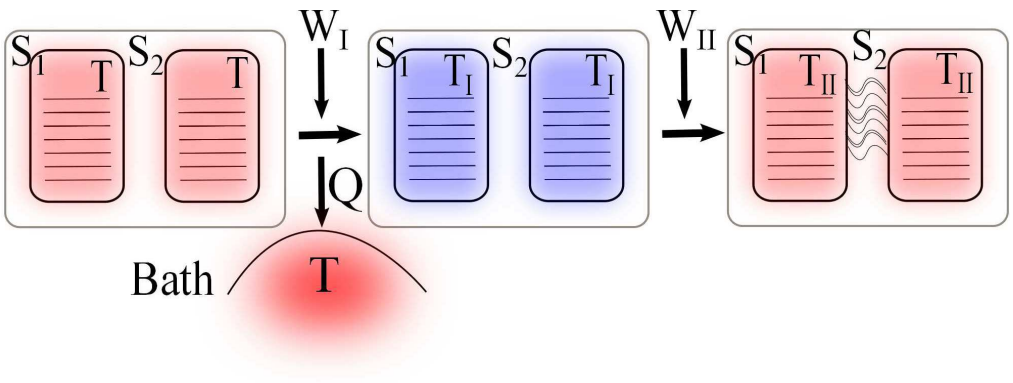

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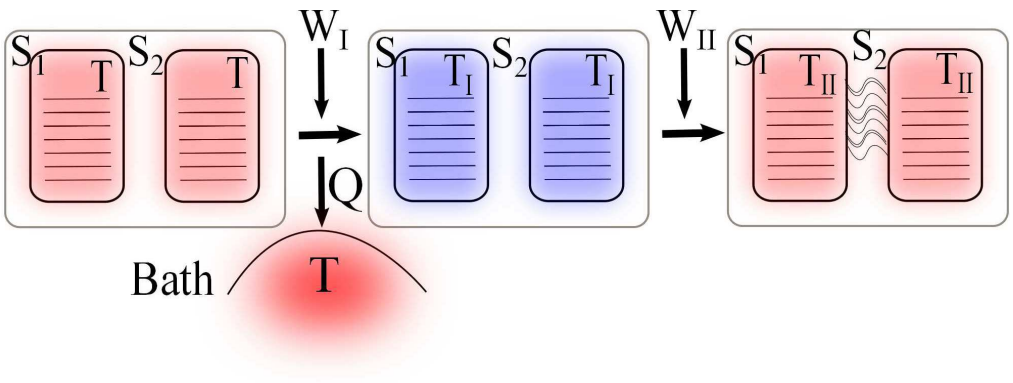

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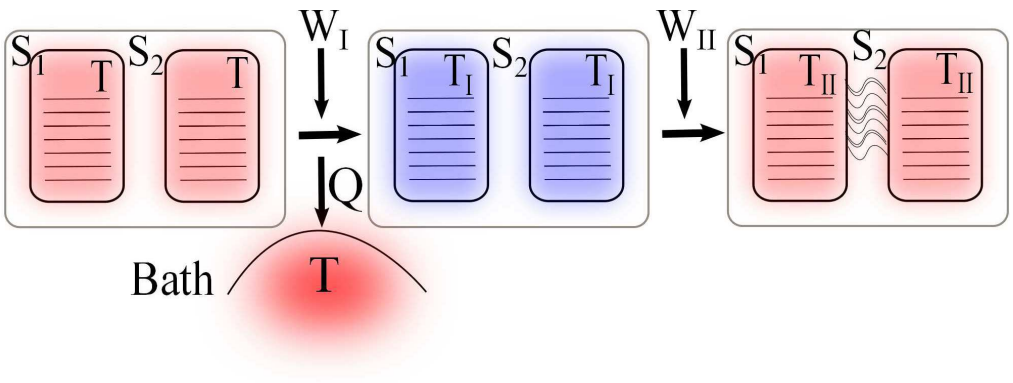

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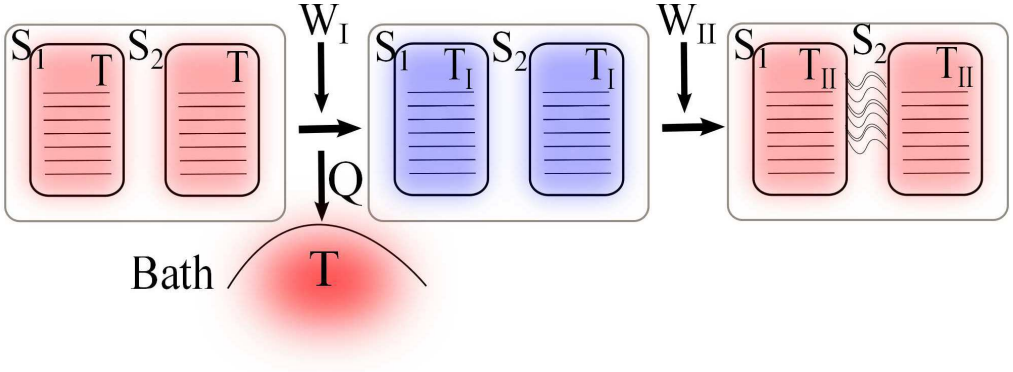

Overall work cost:

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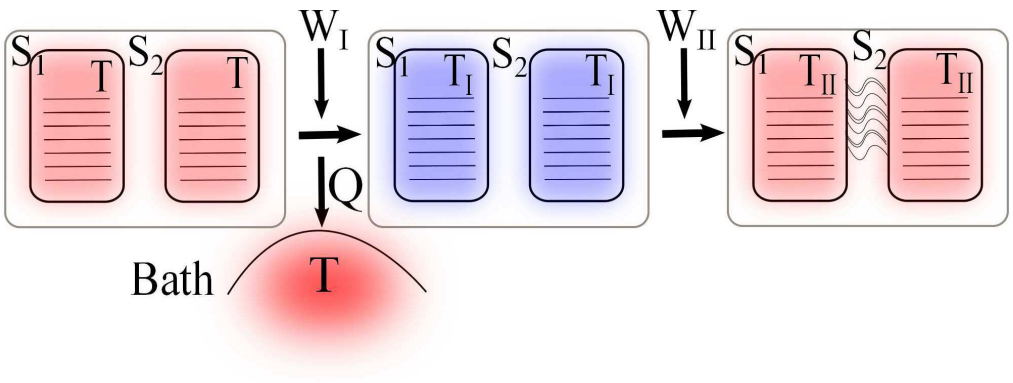

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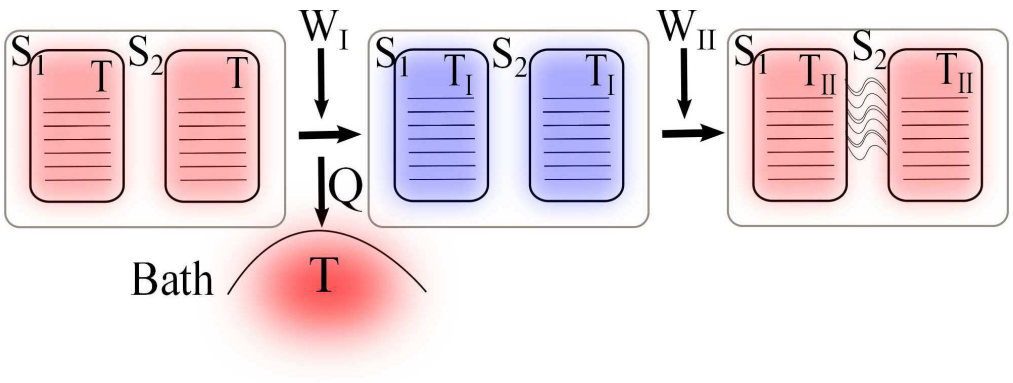

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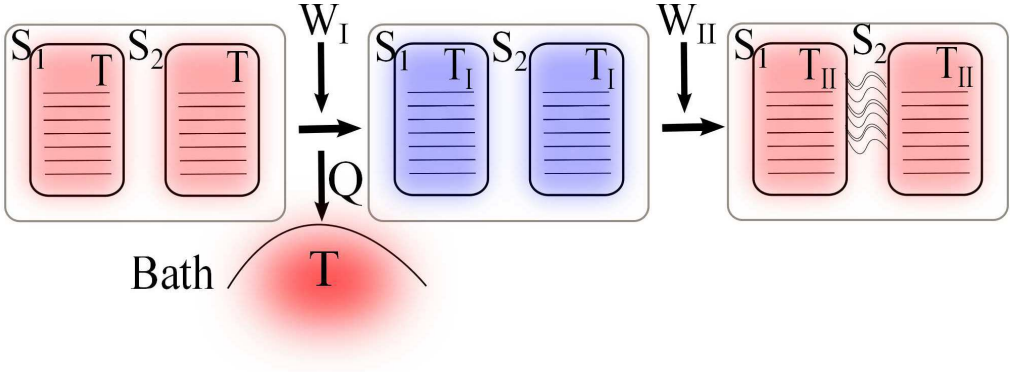

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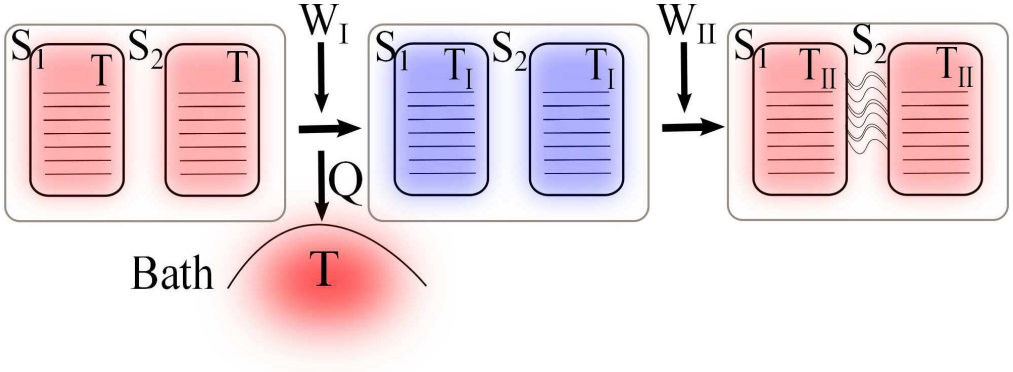

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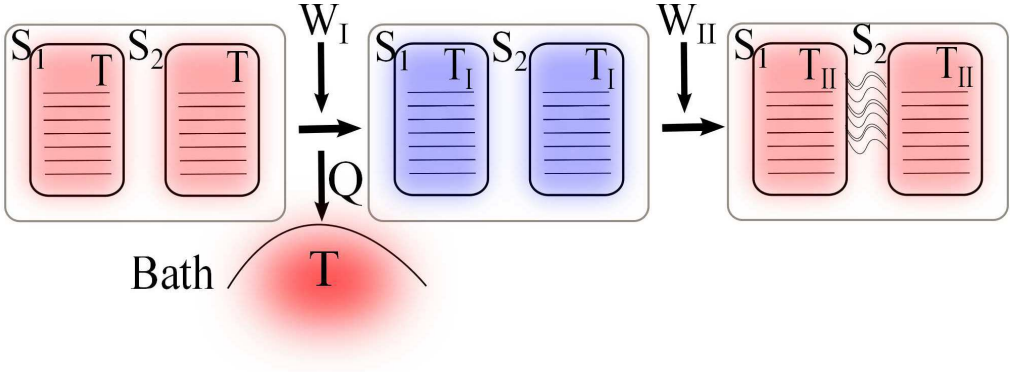

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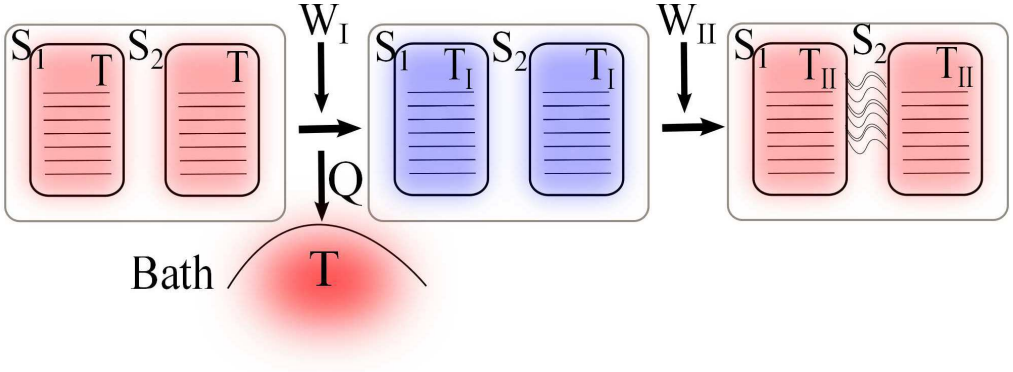

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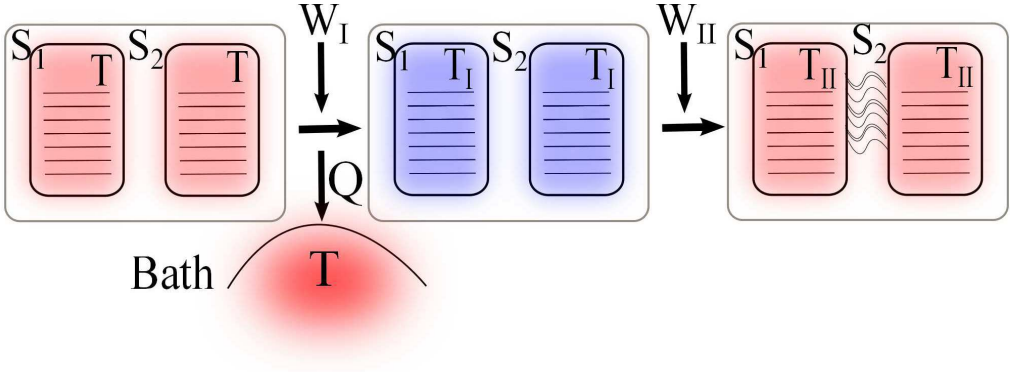

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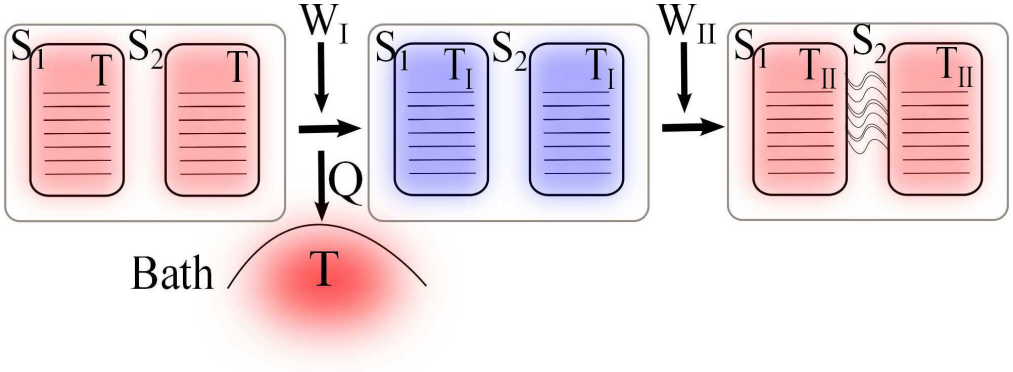

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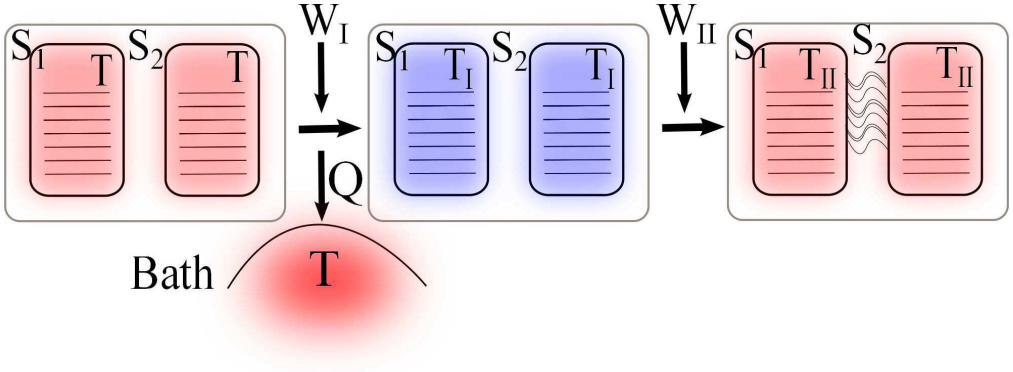
$$

Optimal protocol for the generation of mutual information

Without loss of generality we may split the protocol into two steps



Overall work cost:

$$
\begin{aligned}
W &= W_{\mathrm{I}} + W_{\mathrm{II}} \\
&= E(\tau_S(\beta_{\mathrm{II}})) - E(\tau_S(\beta)) - T\left[S(\tau_S(\beta_{\mathrm{II}})) - S(\tau_S(\beta))\right] + T\,\mathcal{I}_{S_1 S_2}
\end{aligned}
$$

On the other hand

$$
S(\tau_S(\beta_{\mathrm{I}})) = S(\tau_S(\beta_{\mathrm{II}})) - \mathcal{I}_{S_1 S_2}
$$

Optimal protocol for the generation of mutual information

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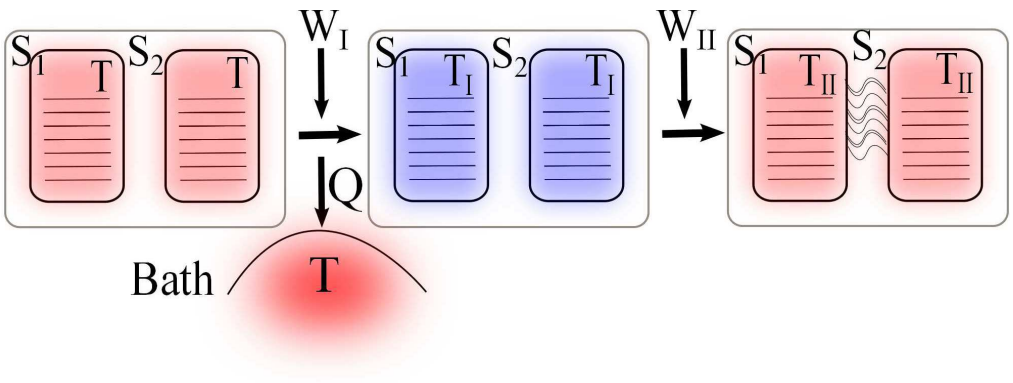

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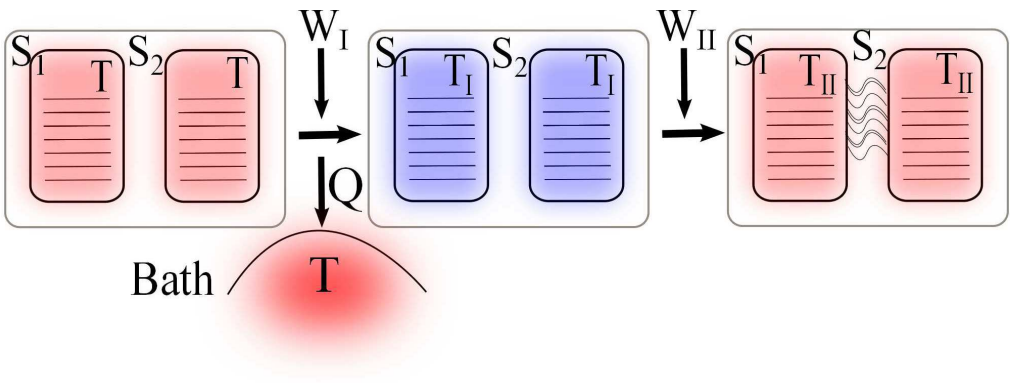
&= \Delta F_S(\beta \to \beta_{\mathrm{II}}) + T\mathcal{I}_{S_1 S_2}
\end{aligned}
$$

Optimal protocol for the generation of mutual information

Without loss of generality we may split the protocol into two steps



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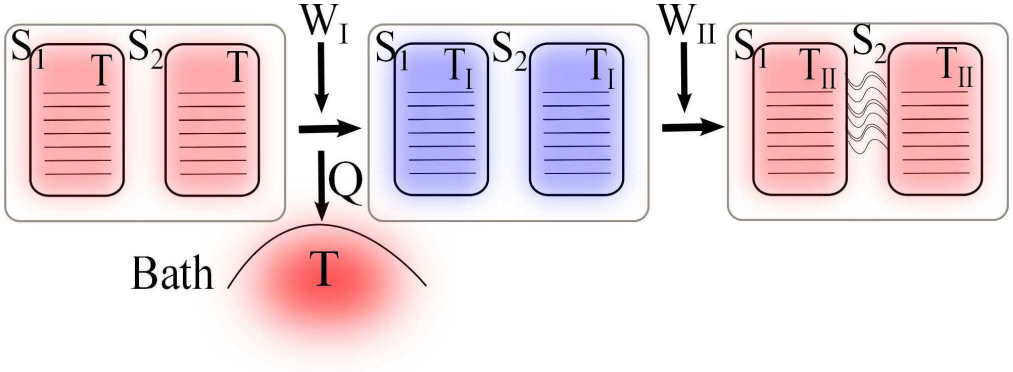

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&= T\,S\big(\tau_S(\beta_{\mathrm{II}}) \| \tau_S(\beta)\big) + T\,\mathcal{I}_{S_1 S_2}
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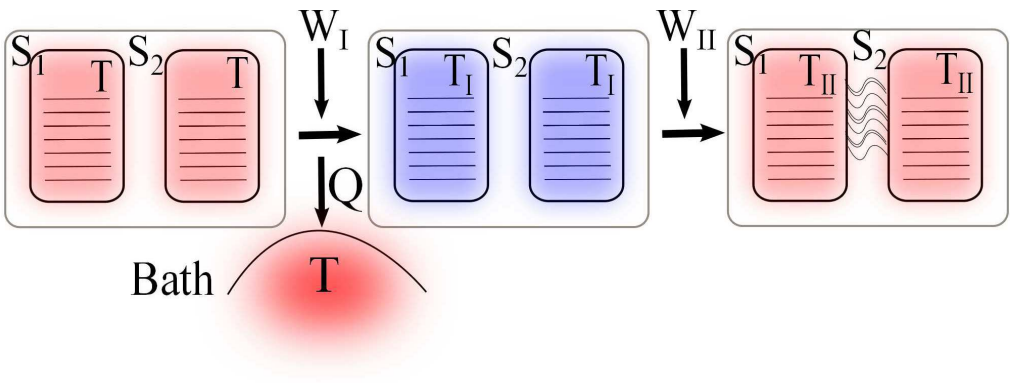

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&= T\,S\big(\tau_S(\beta_{\mathrm{II}}) \| \tau_S(\beta)\big) + T\,\mathcal{I}_{S_1 S_2}
\end{aligned}
$$

Optimal conversion of energy into correlations when

$$\beta_{\mathrm{II}} = \beta, \;\; T_{\mathrm{II}} = T \quad \Rightarrow \quad S\big(\tau_S(\beta_{\mathrm{II}}) \| \tau_S(\beta)\big) = 0$$

Optimal protocol for the generation of mutual information

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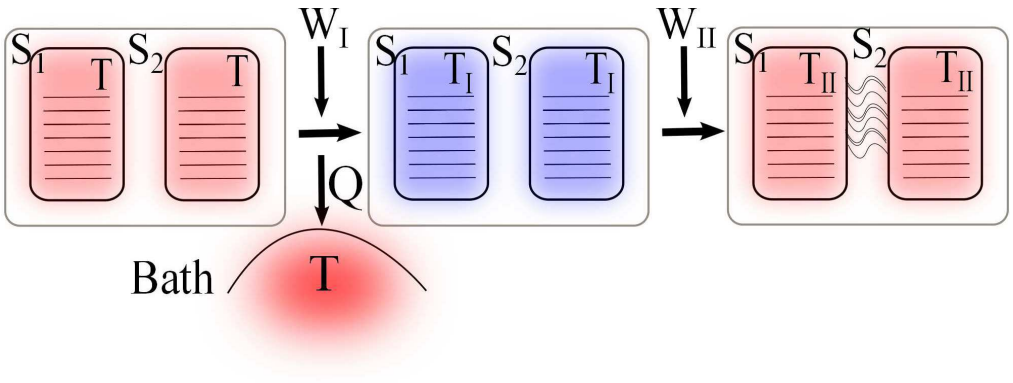

Optimal protocol: distinguish two regimes

Low energy: $T_{\mathrm{II}} = T, \quad T_{\mathrm{I}} > 0,$

High energy: $T_{\mathrm{II}} > T, \quad T_{\mathrm{I}} = 0,$ ground state reached in step I

Threshold energy: $W = T\,S(\tau_S(\beta)) = -F(\tau_S(\beta)) + E(\tau_S(\beta))$

$W_{\mathrm{I}} = -F(\tau_S(\beta)) \geq 0$ free energy difference to ground state

$W_{\mathrm{II}} = E(\tau_S(\beta))$ supplies energy for $\beta_{\mathrm{II}} = \beta$

Optimal protocol for the generation of mutual information

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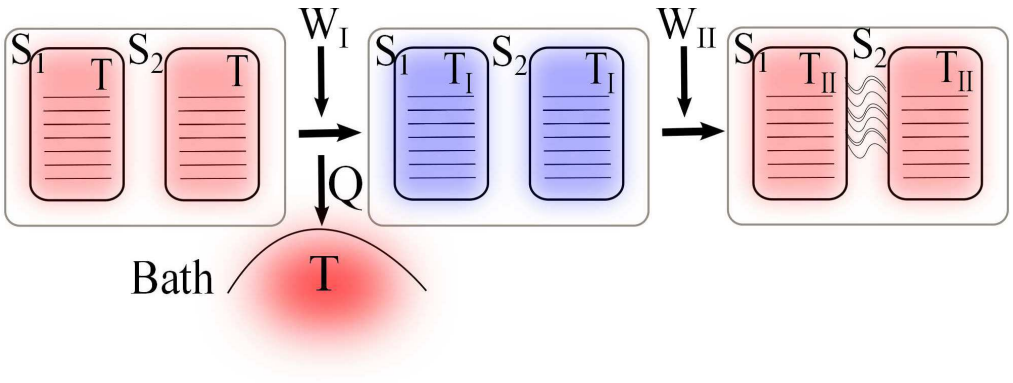

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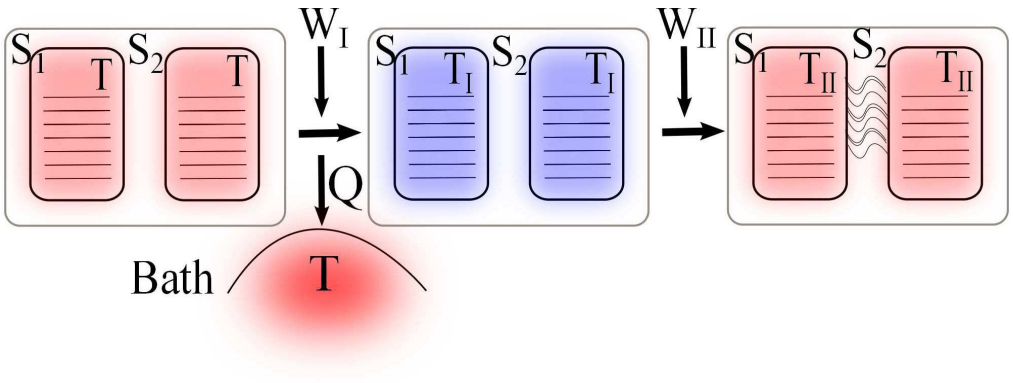

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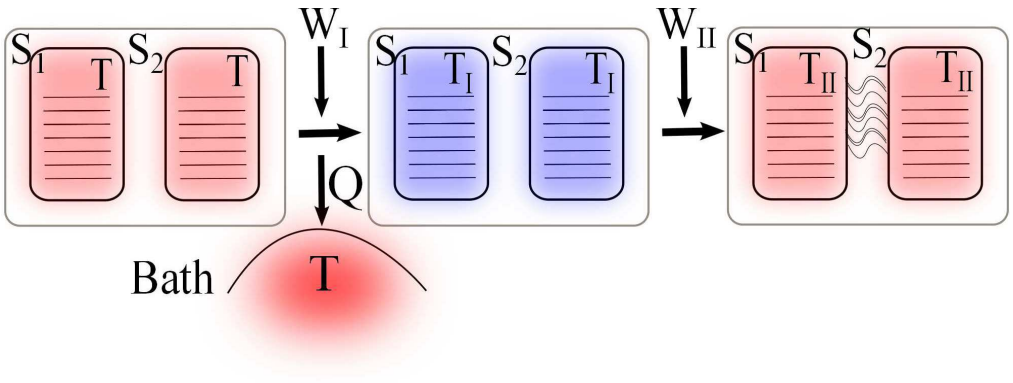

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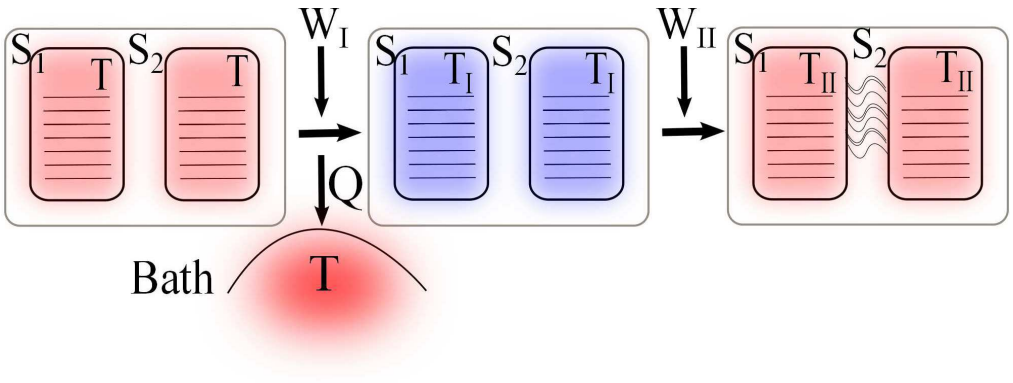

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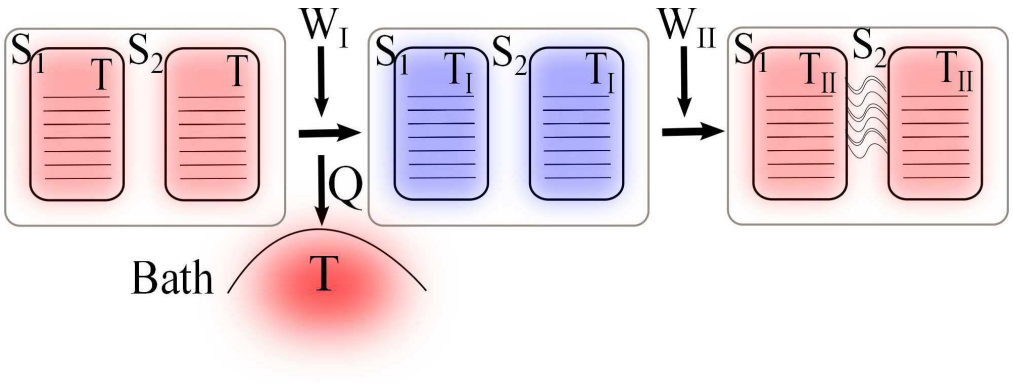

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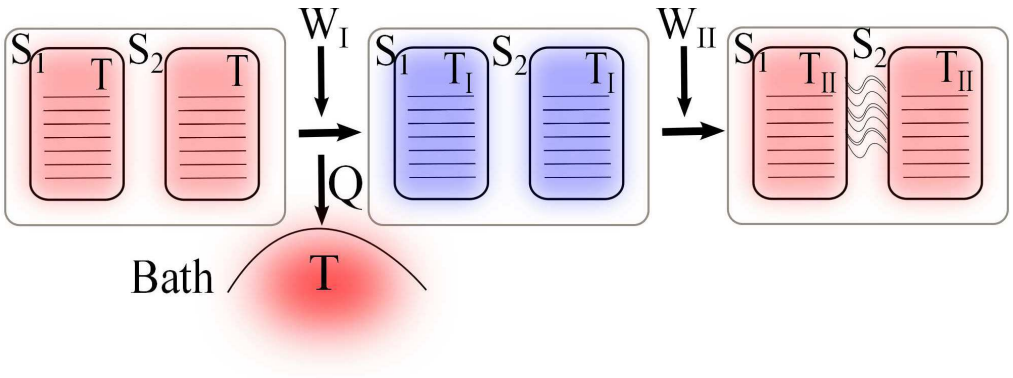

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Low energy: $\mathcal{I}_{S_1 S_2} = \beta W$ if $W \leq T\, S(\tau_S(\beta))$,

High energy: $\mathcal{I}_{S_1 S_2} = S(\tau_S(\beta_{\rm II}))$ if $W > T\, S(\tau_S(\beta))$,

$\beta_{\rm II}$ implicitly given by $E(\tau_S(\beta_{\rm II})) = W + F(\tau_S(\beta))$

Optimal protocol for the generation of mutual information

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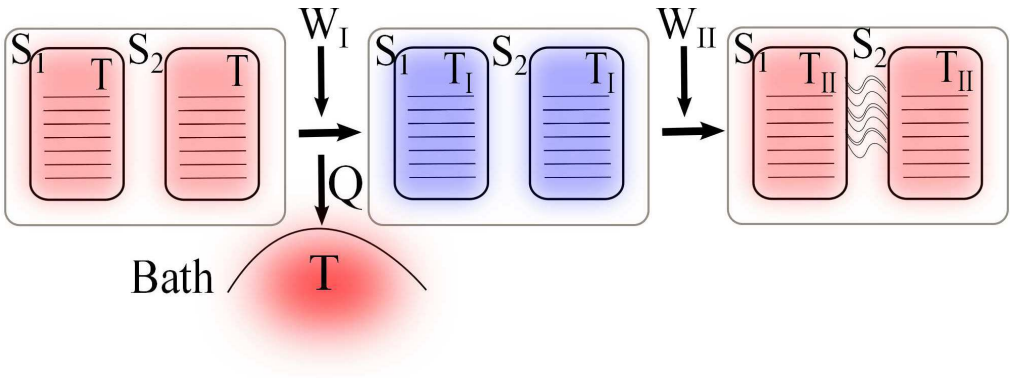

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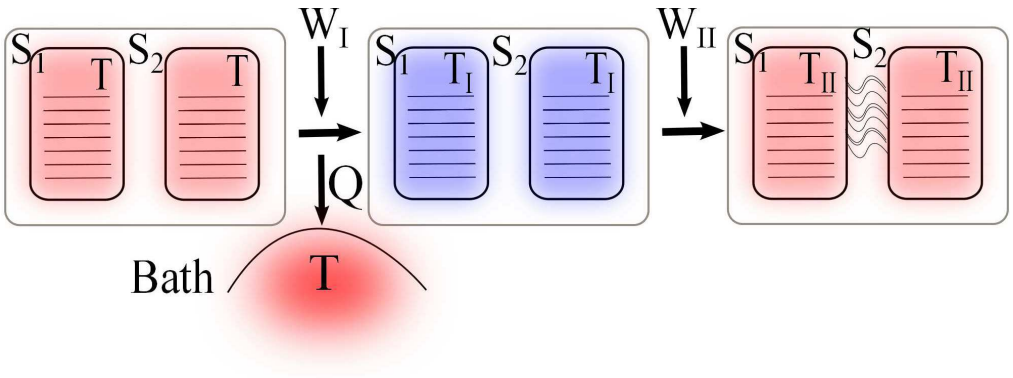

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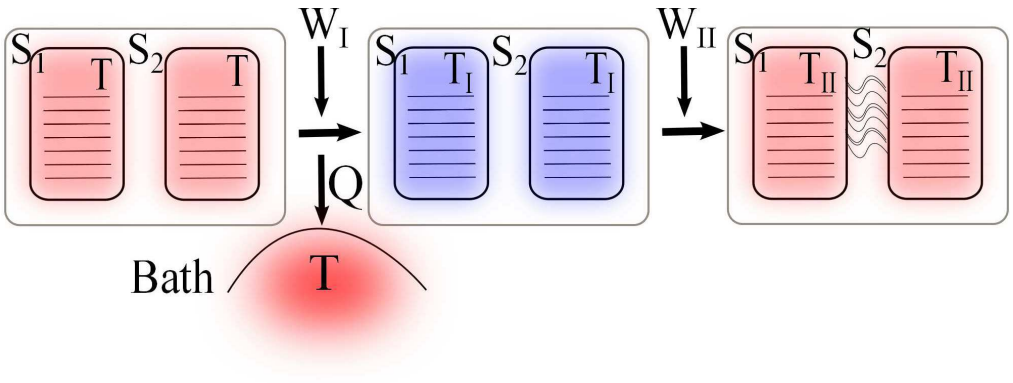

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High energy regime: $W > T\,S(\tau_S(\beta))$

2 fermionic modes at frequency ω

Fermi-Dirac statistics: partition function $\mathcal{Z}_{\mathrm{FD}} = 1 + e^{-\beta}$

Thermal state: average particle number $N_{S_1} = N_{S_2} = \left(e^{\beta} + 1\right)^{-1}$

Pauli exclusion principle: $0 \leq N_{S_i} \leq 1$ for any state

$\Rightarrow$ maximally useful energy for protocol: $W_{\max} = 2T \ln\left(e^{\beta} + 1\right) - \omega$

Notation: $\beta = \omega/T$, temperatures specified in units of $\hbar\omega/k_{\mathrm{B}}$, $T = 0,\ 0.1, \ldots, 0.9,\ 1$

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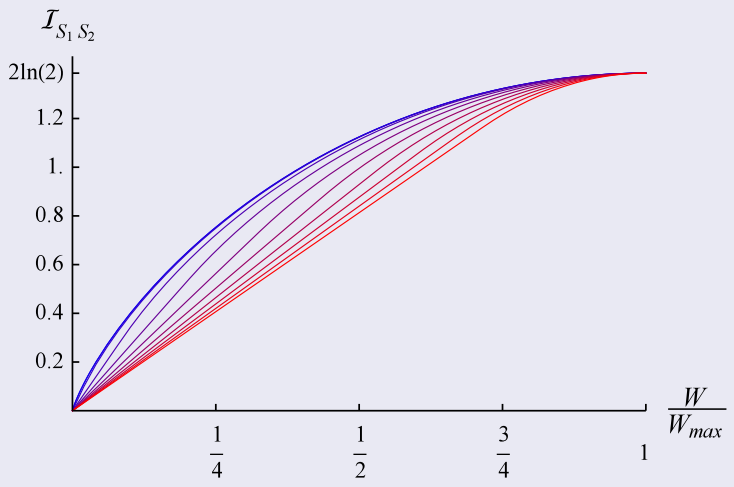

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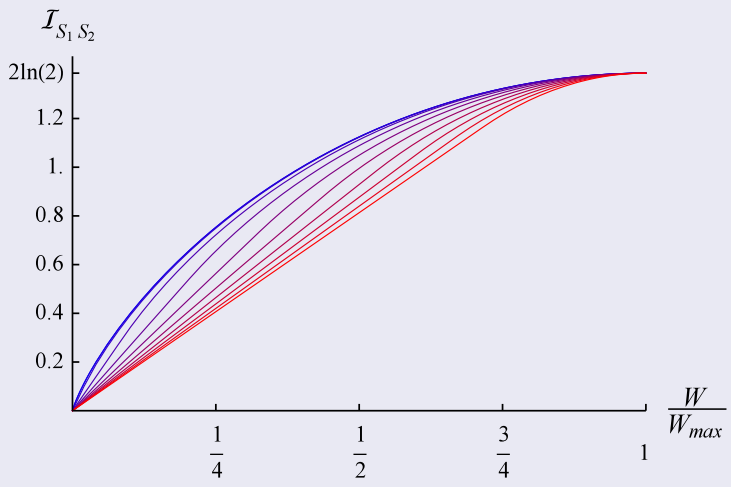

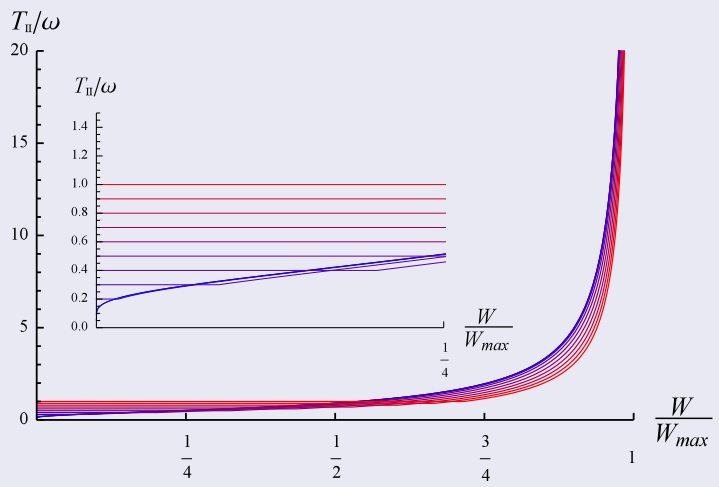

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2 bosonic modes at frequency ω

Bose-Einstein statistics: partition function $\mathcal{Z}_{\mathrm{BE}} = \left(1 - e^{-\beta}\right)^{-1}$

Thermal state: average particle number $N_{S_1} = N_{S_2} = \left(e^{\beta} - 1\right)^{-1}$

Energy of thermal state: $E\big(\tau_S(\beta)\big) = \omega\Big[\coth\big(\beta/2\big) - 1\Big]$

Entropy of thermal state: $S\big(\tau_S(\beta)\big) = 2\,f\Big(\coth\big(\beta/2\big)\Big)$

where $f(x) = \frac{x+1}{2}\ln\left(\frac{x+1}{2}\right) - \frac{x-1}{2}\ln\left(\frac{x-1}{2}\right)$

For $W \gg T\,S(\tau_S(\beta))$: all energy of step II for correlations

$W_{\mathrm{II}} = E\big(\tau_S(\beta_{\mathrm{II}})\big)$ and $\mathcal{I}_{S_1S_2} = S\big(\tau_S(\beta_{\mathrm{II}})\big)$

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$$\mathcal{I}_{S_1S_2} = 2 + 2\ln\left(\frac{1}{2}\frac{W_{\mathrm{II}}}{\omega}\right) + \mathcal{O}\left(\frac{\omega}{W_{\mathrm{II}}}\right)$$

Notation: $\beta = \omega/T$, temperatures specified in units of $\hbar\omega/k_{\mathrm{B}}$

High energy regime: $W > T\,S(\tau_S(\beta))$

2 bosonic modes at frequency ω

Bose-Einstein statistics: partition function $\mathcal{Z}_{\mathrm{BE}} = \left(1 - e^{-\beta}\right)^{-1}$

Thermal state: average particle number $N_{S_1} = N_{S_2} = \left(e^{\beta} - 1\right)^{-1}$

Energy of thermal state: $E(\tau_S(\beta)) = \omega\left[\coth(\beta/2) - 1\right]$

Entropy of thermal state: $S(\tau_S(\beta)) = 2f\Big(\coth(\beta/2)\Big)$

where $f(x) = \frac{x+1}{2}\ln\left(\frac{x+1}{2}\right) - \frac{x-1}{2}\ln\left(\frac{x-1}{2}\right)$

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Optimal creation of entanglement

Entanglement of Formation for two fermionic modes

Superselected entanglement of formation

$$E_{oF} = \inf_{\mathcal{D}(\rho_S)} \sum_i p_i \, \mathcal{E}\big(|\,\psi_i\,\rangle\big)$$

where $\mathcal{E}\big(|\,\psi_i\,\rangle\big) = S\Big(\mathrm{Tr}_{S_2}\big(|\,\psi_i\,\rangle\langle\,\psi_i\,|\big)\Big)$

and $\mathcal{D}(\rho_S) = \big\{p_i, |\,\psi_i\,\rangle \,|\, \sum_i p_i \,|\,\psi_i\,\rangle\langle\,\psi_i\,| = \rho_S\big\}$

superselection rules: take infimum only over allowed states $|\,\psi_i\,\rangle$

Here: no superpositions of even and odd fermions numbers

With this: even and odd subspace decouple: θ_{even} and θ_{odd}

$$E_{oF} = \ln(2)\big[|1 - N^{\mathrm{I}}_{S_1} - N^{\mathrm{I}}_{S_2}|\sin(2\theta_{\mathrm{even}}) + |N^{\mathrm{I}}_{S_1} - N^{\mathrm{I}}_{S_2}|\sin(2\theta_{\mathrm{odd}})\big]$$

energy cost: $\frac{W_{\mathrm{II}}}{\omega} = 2\big(1 - N^{\mathrm{I}}_{S_1} - N^{\mathrm{I}}_{S_2}\big)\sin^2(\theta_{\mathrm{even}})$

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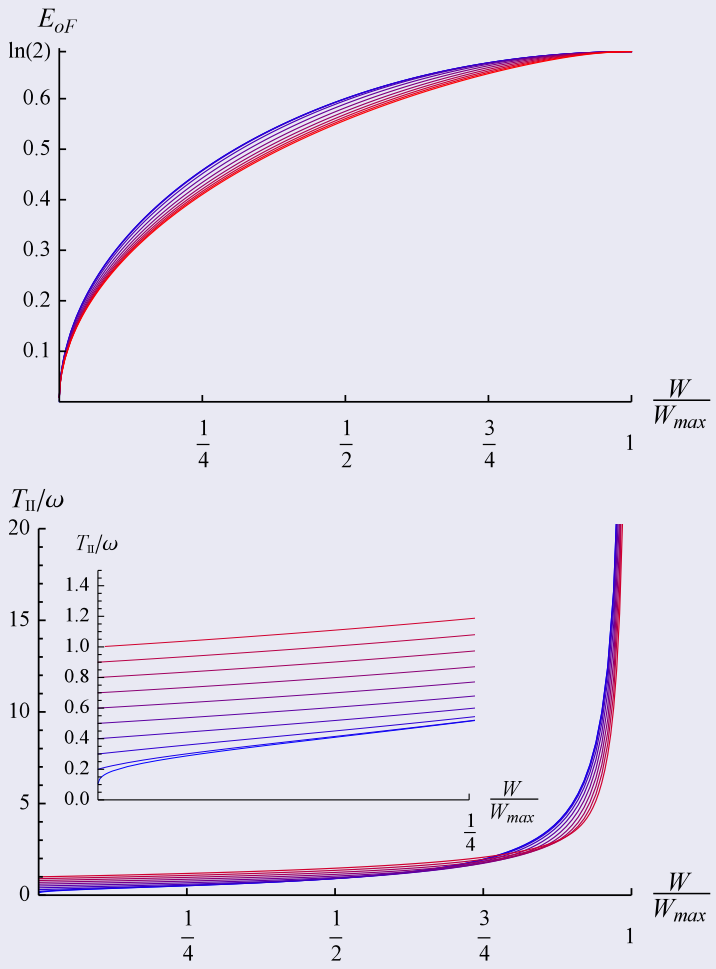
Previous Protocol



Optimal Protocol

Notation: $\beta = \omega/T$, temperatures specified in units of $\hbar\omega/k_{\mathrm{B}}$, $T = 0,\ 0.1, \ldots, 0.9,\ 1$ and ∞

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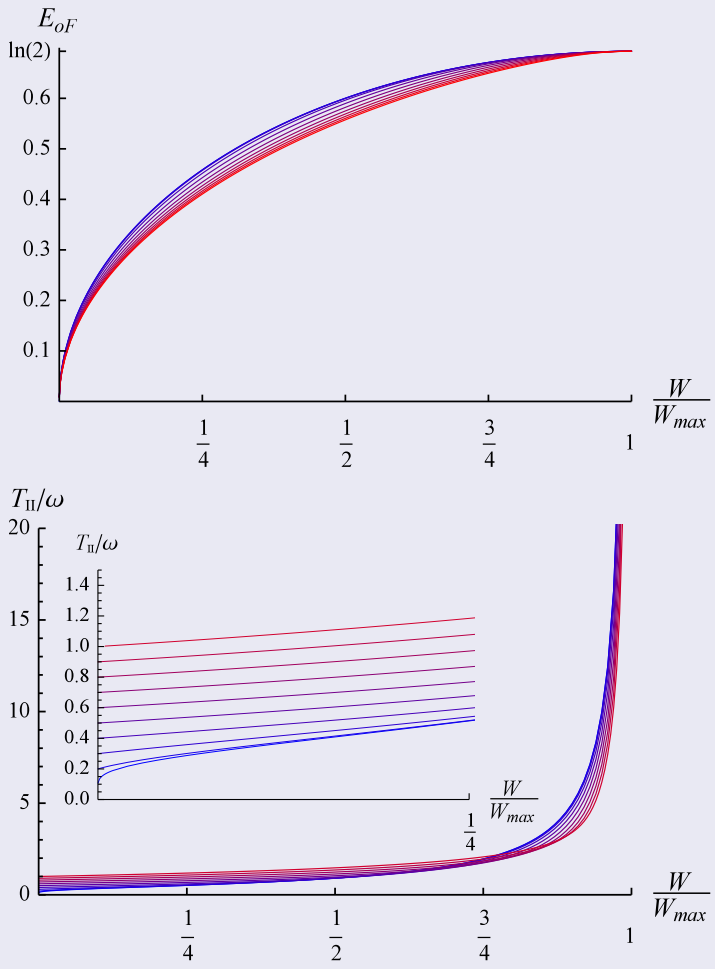

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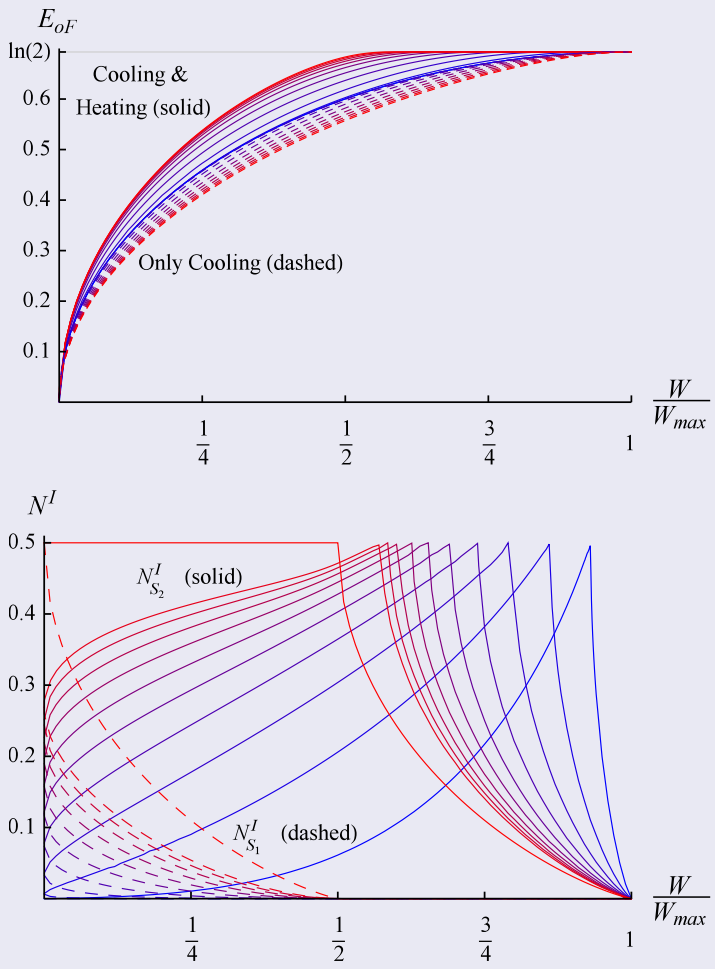

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State description for two bosonic modes

two harmonic oscillators: Hilbert space infinite-dimensional $\Rightarrow$ restrict problem

Two-mode Gaussian states

Fully described by 1st and 2nd moments: expectations values of quadratures $\mathbb{X}_n$

$$\mathbb{X}_{(2n-1)} = \frac{1}{\sqrt{2}}(a_n + a_n^\dagger) \quad \text{and} \quad \mathbb{X}_{(2n)} = \frac{-i}{\sqrt{2}}(a_n - a_n^\dagger)$$

First moments: $\mathrm{Tr}\big(\mathbb{X}_n \rho_S\big) = 0$ irrelevant for entanglement

Second moments: $(\sigma_S)_{mn} = \mathrm{Tr}\big(\{\,\mathbb{X}_m\,,\mathbb{X}_n\,\}\,\rho_S\big)$ **covariance matrix**

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Energy for cooling: $\frac{W_{\mathrm{I}}}{\omega} = \nu(T_{\mathrm{I}}) - \nu(T) - 2\beta^{-1}\Big[f\big(\nu(T_{\mathrm{I}})\big) - f\big(\nu(T)\big)\Big]$

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Notation: $\beta = \omega/T$, temperatures specified in units of $\hbar\omega/k_{\mathrm{B}}$

State description for two bosonic modes

two harmonic oscillators: Hilbert space infinite-dimensional $\Rightarrow$ restrict problem

Two-mode Gaussian states

Fully described by 1st and 2nd moments: expectations values of quadratures $\mathbb{X}_n$

$$\mathbb{X}_{(2n-1)} = \frac{1}{\sqrt{2}}(a_n + a_n^\dagger) \quad \text{and} \quad \mathbb{X}_{(2n)} = \frac{-i}{\sqrt{2}}(a_n - a_n^\dagger)$$

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Necessary and sufficient condition: negative partial transpose (NPT)

Translates to: entanglement if and only if $0 \leq \tilde{\nu}_- < 1$,

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Optimal Gaussian entangling operation: state remains symmetric: E_{oF} computable

$$E_{oF} = \begin{cases} \mathfrak{h}(\tilde{\nu}_-)\,, & \text{if } \; 0 \leq \tilde{\nu}_- < 1\,, \\ 0\,, & \text{if } \; \tilde{\nu}_- \geq 1\,, \end{cases}$$

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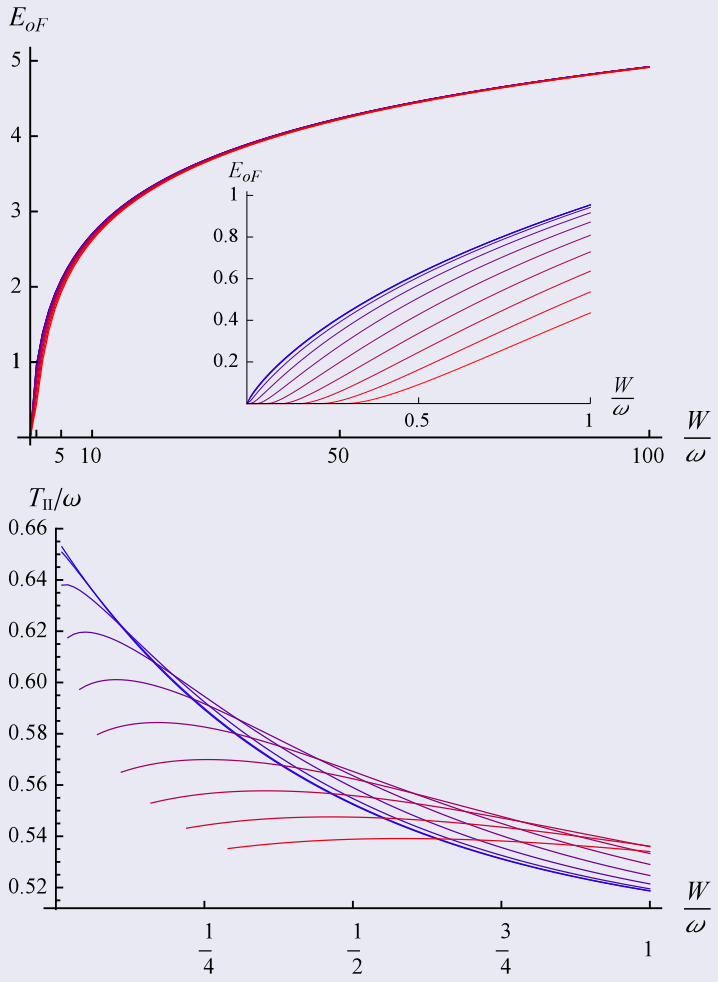

Optimal Protocols

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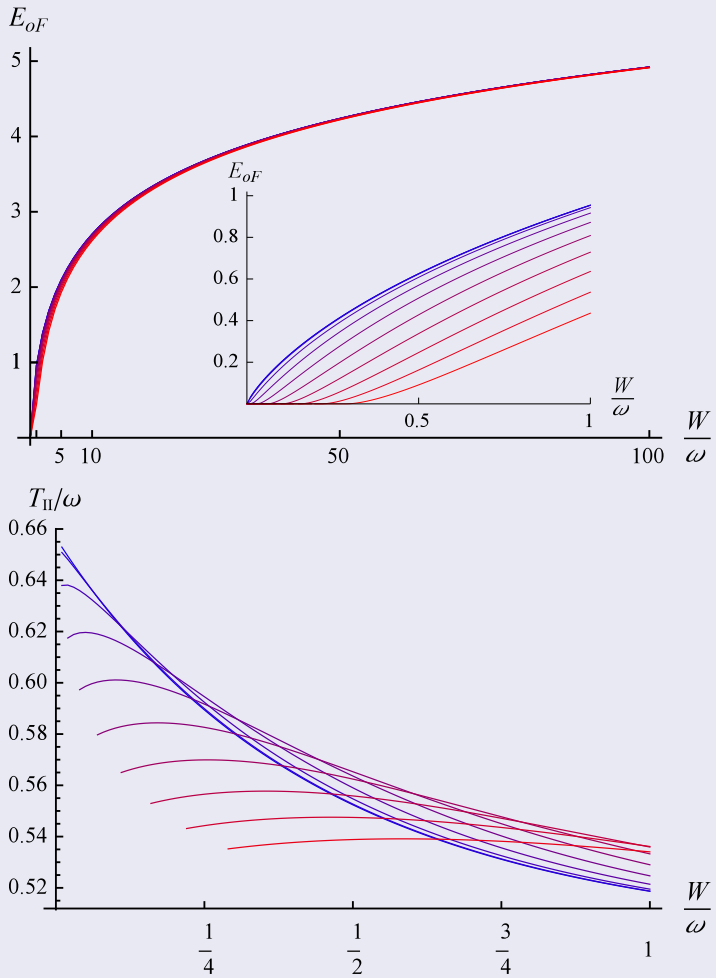

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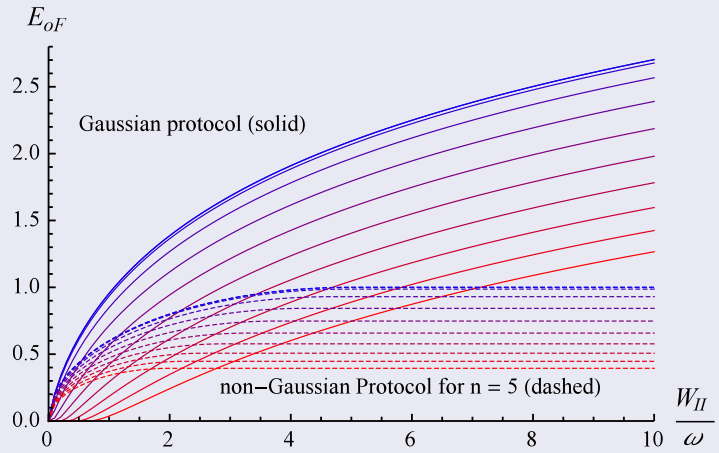

High energies:

Gaussian protocol optimal

Low energies:

Outperformed by non-Gaussian protocol:

Rotation to "Bell state" in 2-dim subspace

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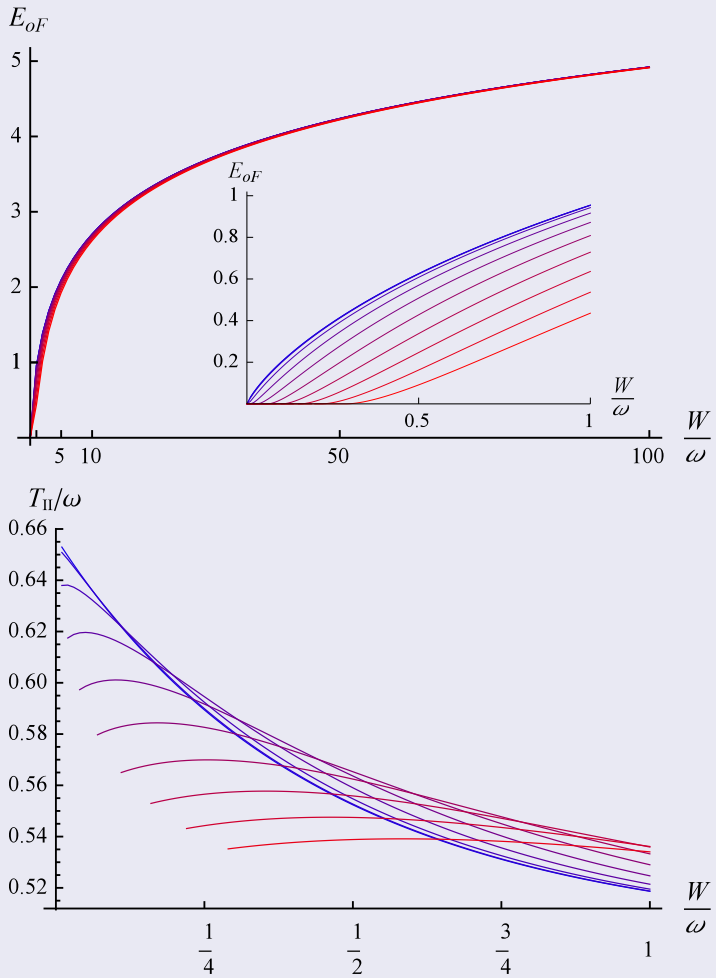

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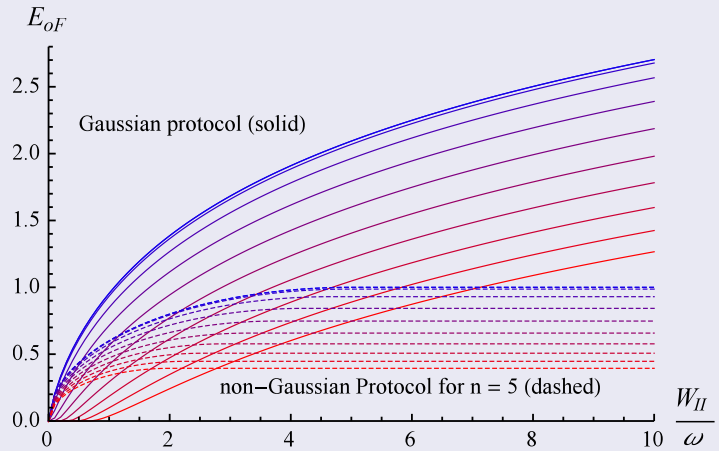

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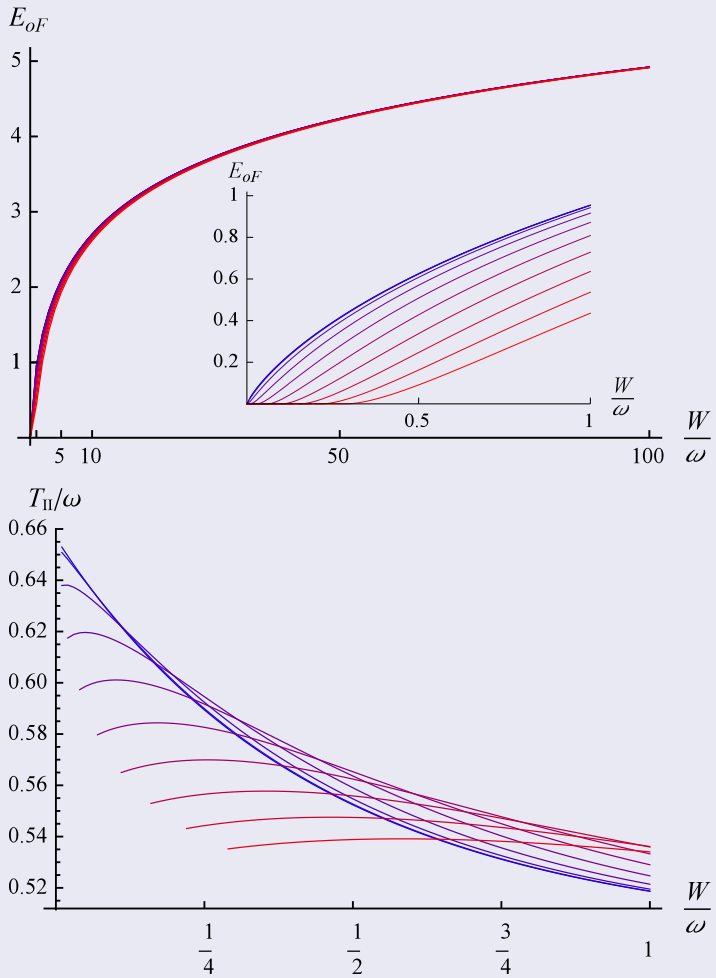

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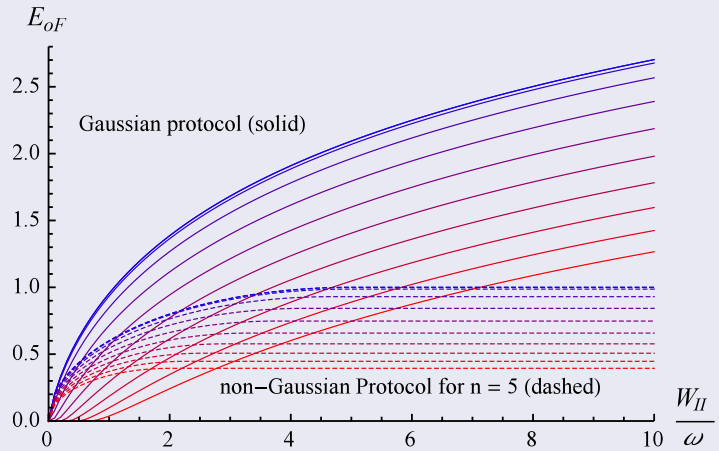

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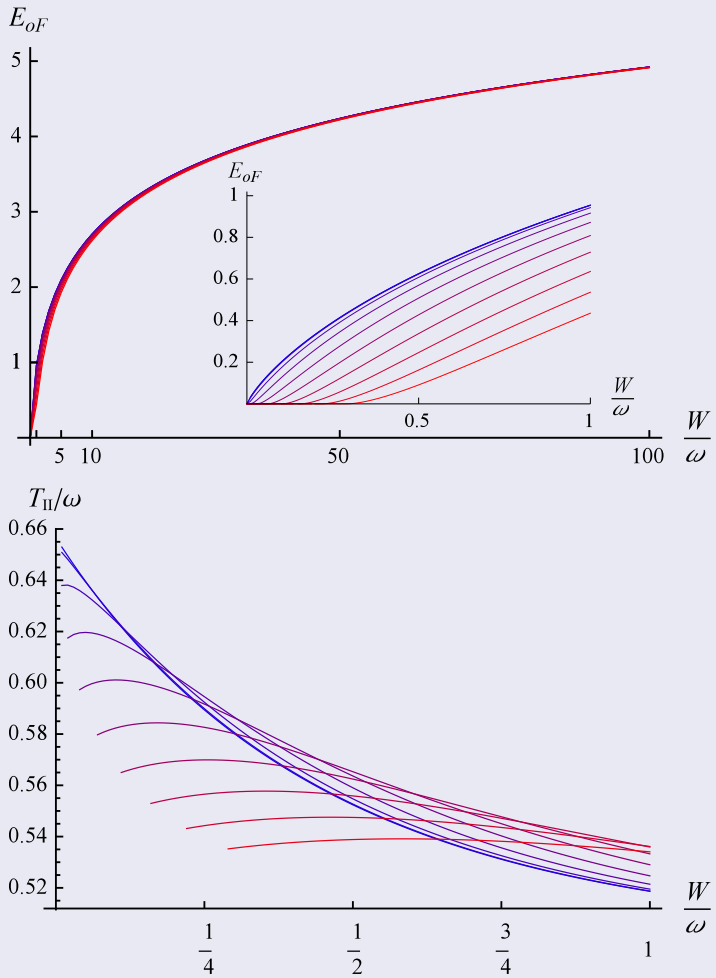

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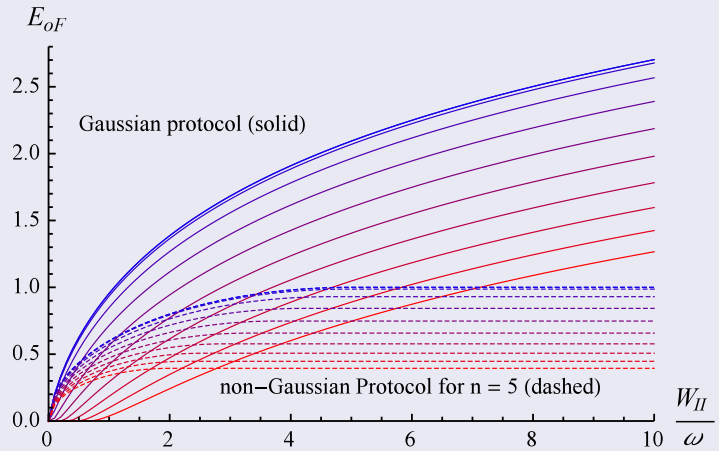

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Thank you for your attention.

D. E. Bruschi, M. Perarnau-Llobet, N. Friis, K. V. Hovhannisyan, and M. Huber, arXiv:1409.4647 [quant-ph].