EPSRC
Engineering and Physical Sciences Research Council

The University of Nottingham

Shaking Entanglement

entanglement generation in relativistic quantum fields

CNQO Seminar

University of Strathclyde, April 25th, 2012

Nicolai Friis - University of Nottingham

work in collaboration with

D. E. Bruschi, I. Fuentes, A. R. Lee and *J. Louko*

Outline

Once upon a time in a Hilbert space far far away . . .

- Introduction to *Relativistic Quantum Information* (RQI)

  Motivation for RQI, entanglement basics,

- Relativistic accelerated motion: Rindler coordinates

- *Quantum Fields* in Cavities

  Bosonic & Fermionic Fields, boundary conditions

- Non-uniform motion: *Bogoliubov* transformations

  $\Rightarrow$ **Effects on Entanglement**

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Relativistic Quantum Information

Motivation

- Resources/Tasks of QI well known:

  How are they affected by *Relativity*?

  Are effects degraded/enhanced?

- Connection between $QFT \Rightarrow QI$

  Unruh effect, *Hawking* effect, *Casimir* effect

- New Ways to ... create, store, transmit, process QI

Our aim here ...

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Entanglement Basics

Pure state entanglement

$\psi \in \mathcal{H}_A \otimes \mathcal{H}_B$ is *separable* iff $|\psi\rangle = |\phi\rangle_A \otimes |\chi\rangle_B$

ψ is *entangled* if it is not separable

Example: Singlet state maximally entangled

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle|1\rangle - |1\rangle|0\rangle\Big)$$

Detection/Quantification

reduced states of subsystems do not contain complete information

$\Rightarrow$ Loss of information $\Rightarrow$ subsystems mixed

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$\Rightarrow$ Loss of information $\Rightarrow$ increase of Entropy

Entanglement Basics

Mixed state entanglement

mixed states $\rho = \sum_i p_i \, |\, \psi_i \,\rangle \langle\, \psi_i \,|$ where $\sum_i p_i = 1$

ρ is *separable* iff $\rho = \sum_i p_i \, \rho_i^A \otimes \rho_i^B$

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Positive Partial Transpose Criterion (Peres, Horodecki, 1996)

$\rho^{T_B} = (\mathbb{1}_A \otimes T_B)\rho \ngeq 0 \quad \Rightarrow \quad \rho$ entangled

Quantification - Negativity $\mathcal{N}$ (Vidal & Werner, 2002)

Eigenvalues λ_i of ρ^{T_B}: $\mathcal{N} = \sum_i \frac{|\lambda_i| - \lambda_i}{2}$

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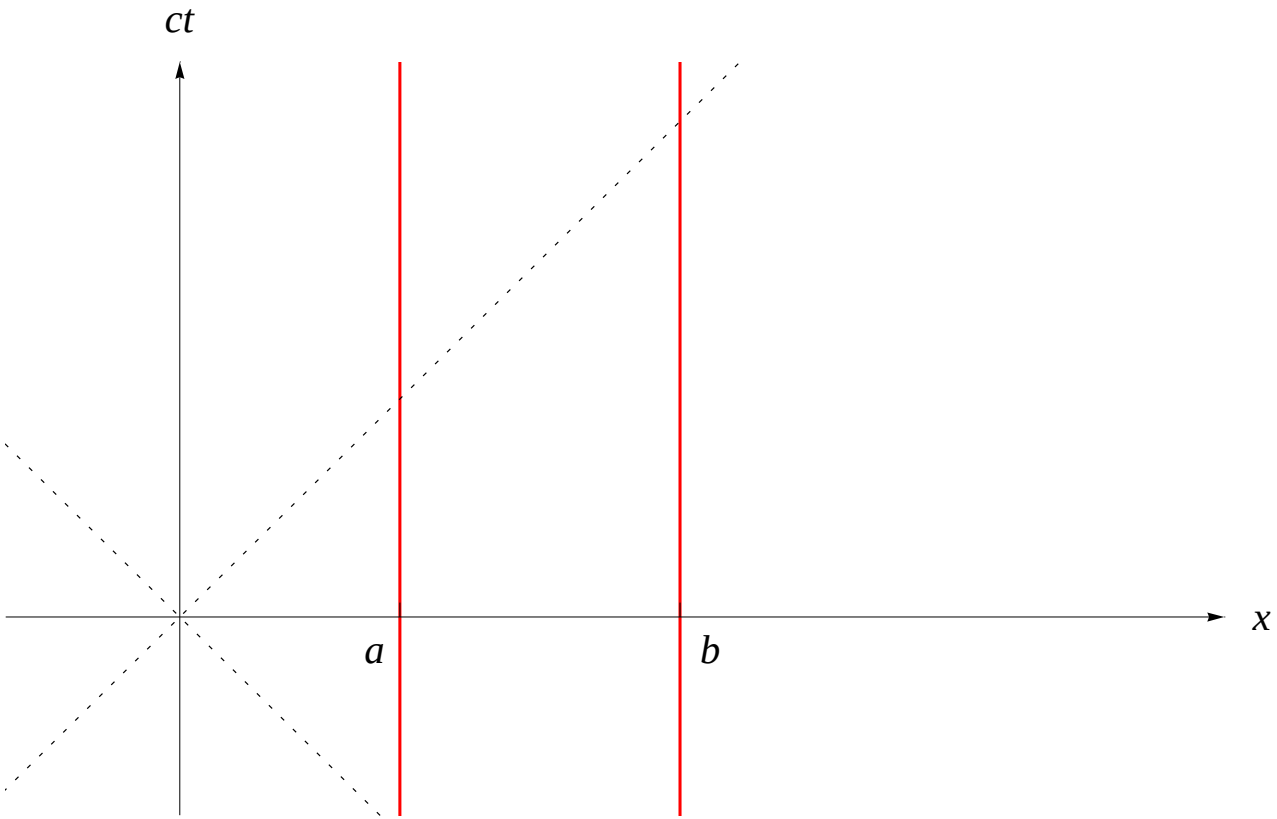
Inertial cavity of width $\delta = b - a$



Uncharged scalar field ϕ

Klein-Gordon Equation: $(\partial^\mu \partial_\mu + m^2)\phi(x) = 0$

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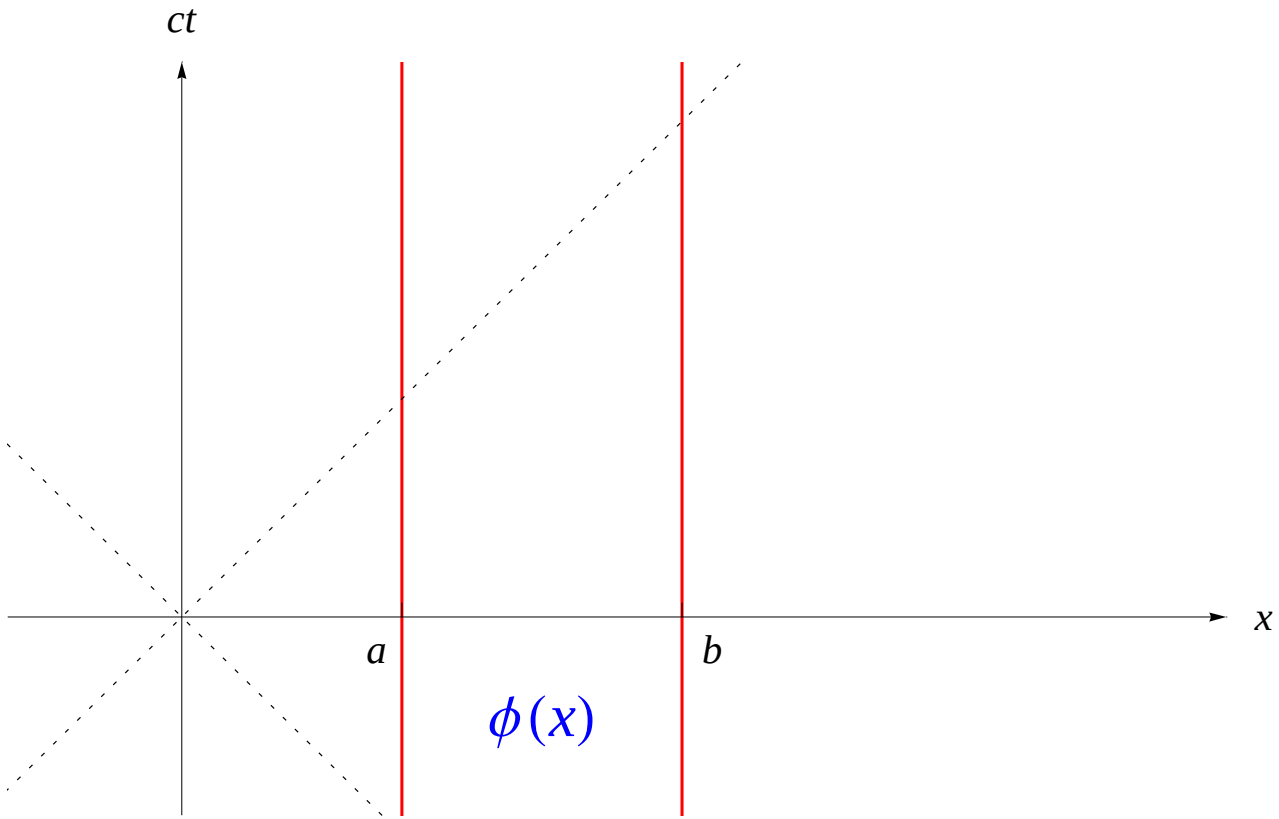

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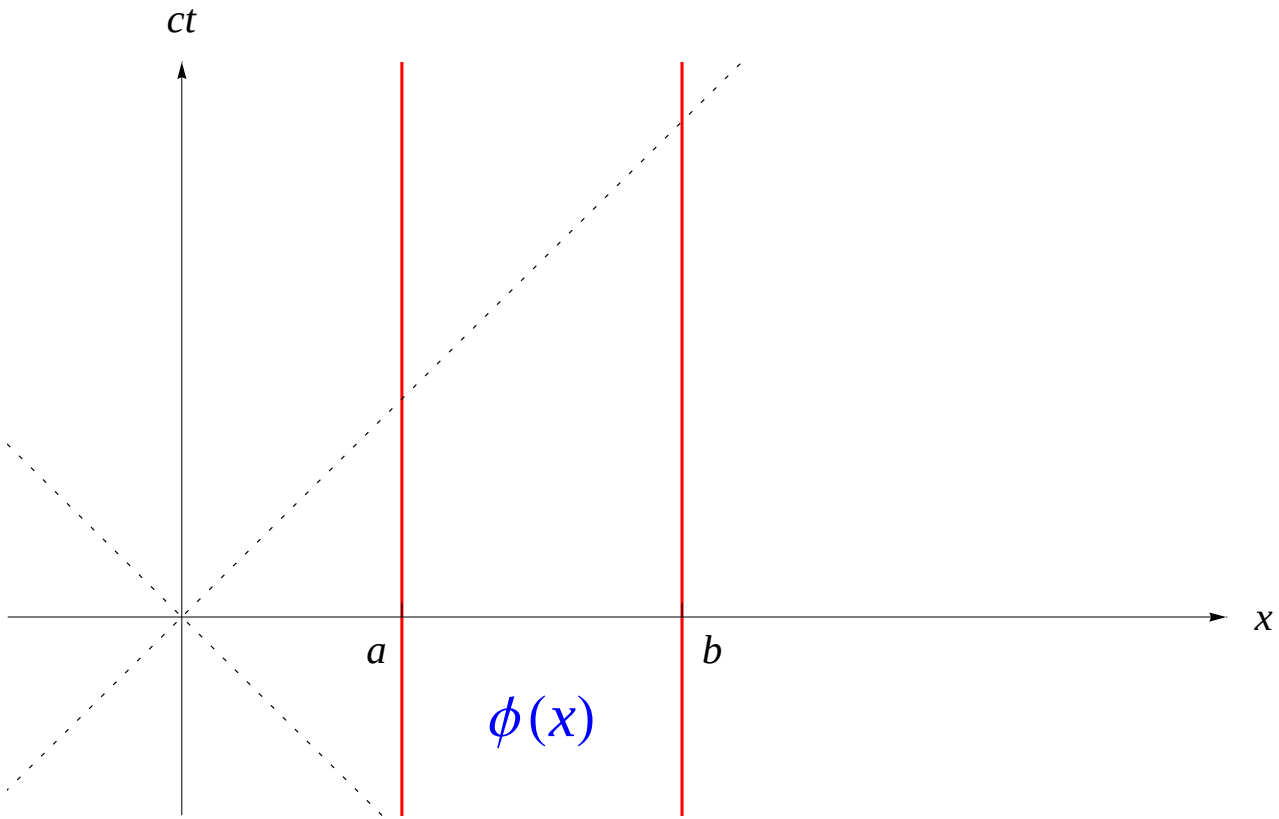

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boundary conditions: *Dirichlet* $\phi(t, \mathbf{a}) = \phi(t, \mathbf{b}) = 0$

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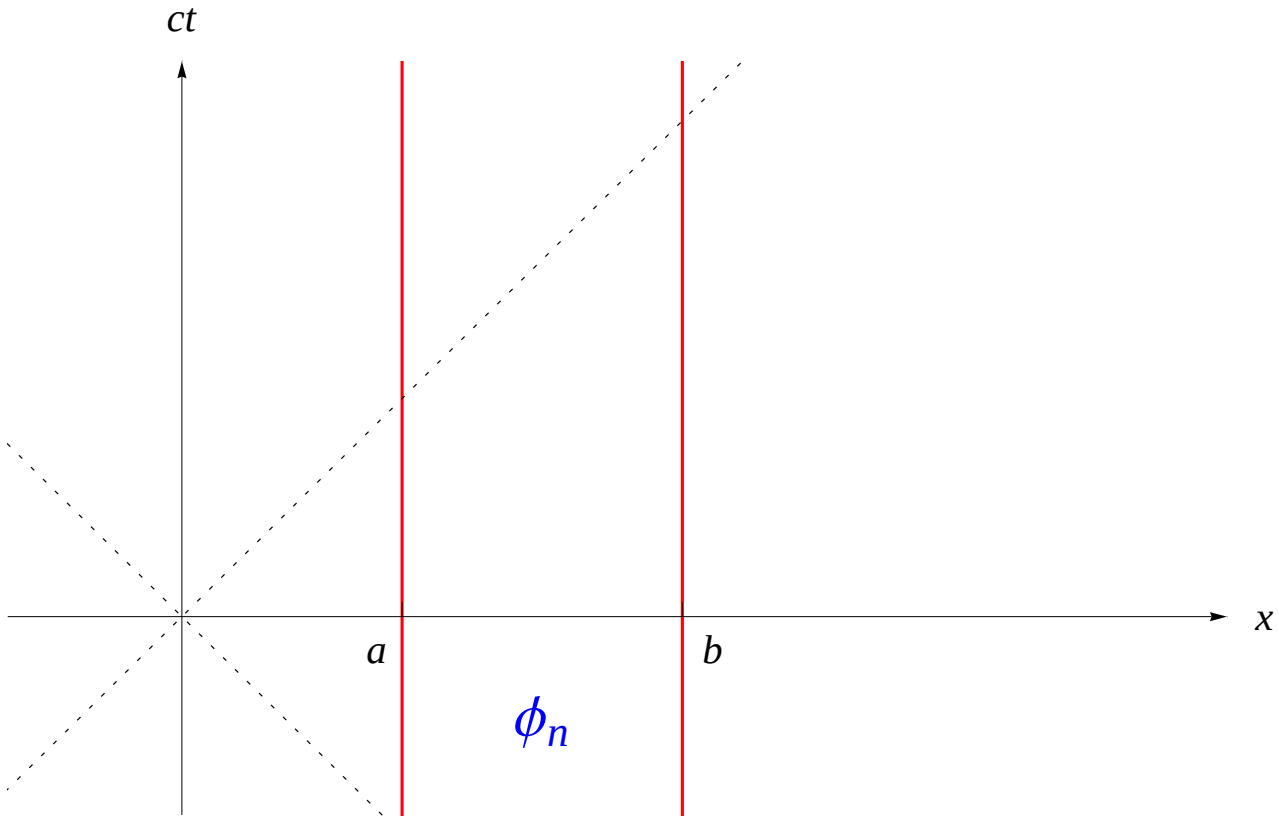

Uncharged scalar field ϕ

solutions: $\{\phi_n\}$: $\phi = \sum_n (\phi_n\, a_n + \phi_n^*\, a_n^\dagger)$, $[\, a_m, a_n^\dagger \,] = \delta_{mn}$

Quantum fields in inertial cavities

Bosons: uncharged scalar

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operators: *commutators*

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Fermions: spin-½

Dirac Equation:

$$(i\gamma^\mu \partial_\mu - m)\,\psi(x) = 0$$

boundaries: *vanishing current*

$$(\bar\psi \gamma^j \psi)_{\mathbf{x}=\mathbf{a},\mathbf{b}} = 0$$

solutions: $\{\psi_n\}$

$$\psi = \sum_n \Big(\psi_n\, b_n + \psi_{-n}\, c_n^\dagger\Big)$$

operators: *anti-commutators*

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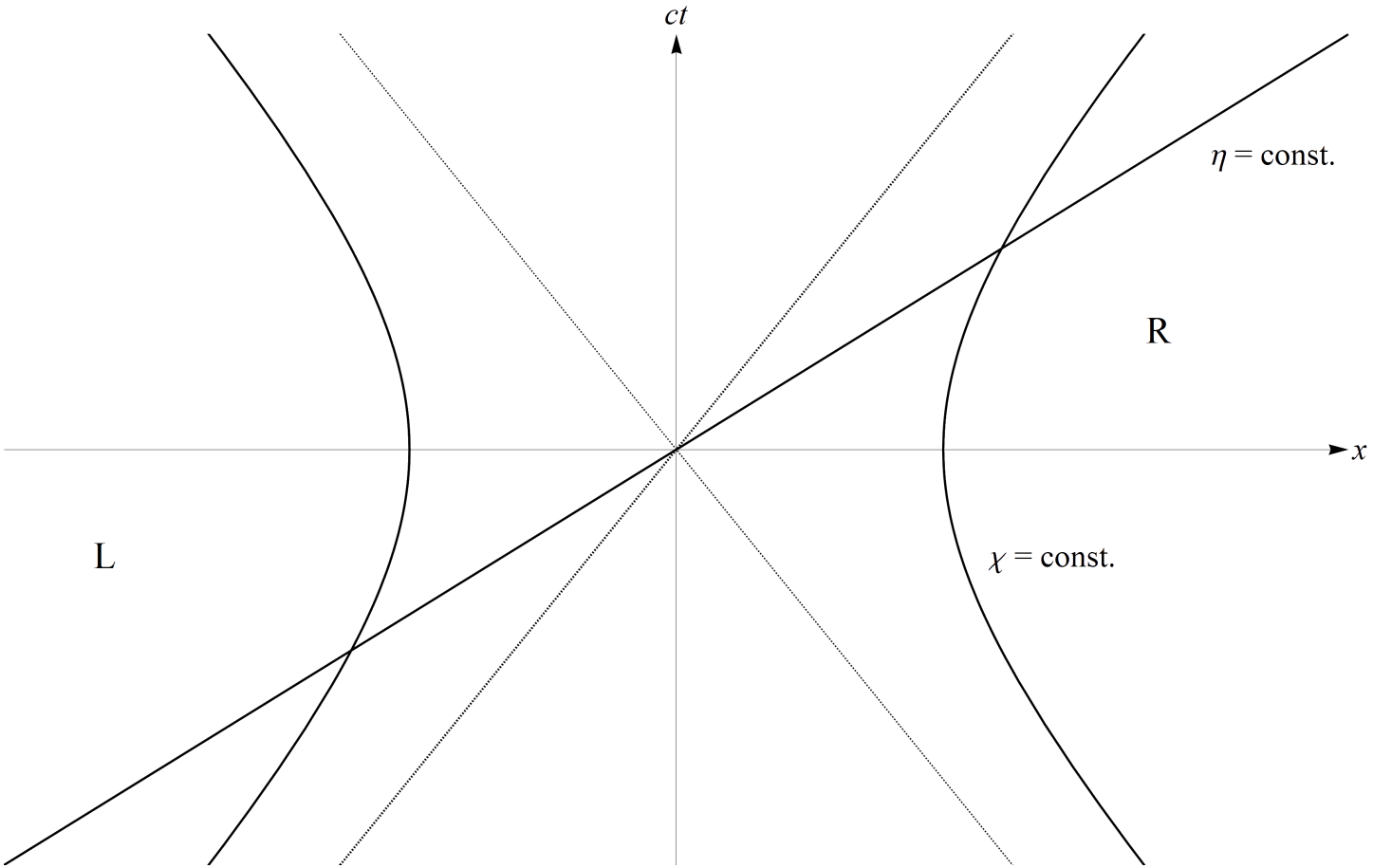
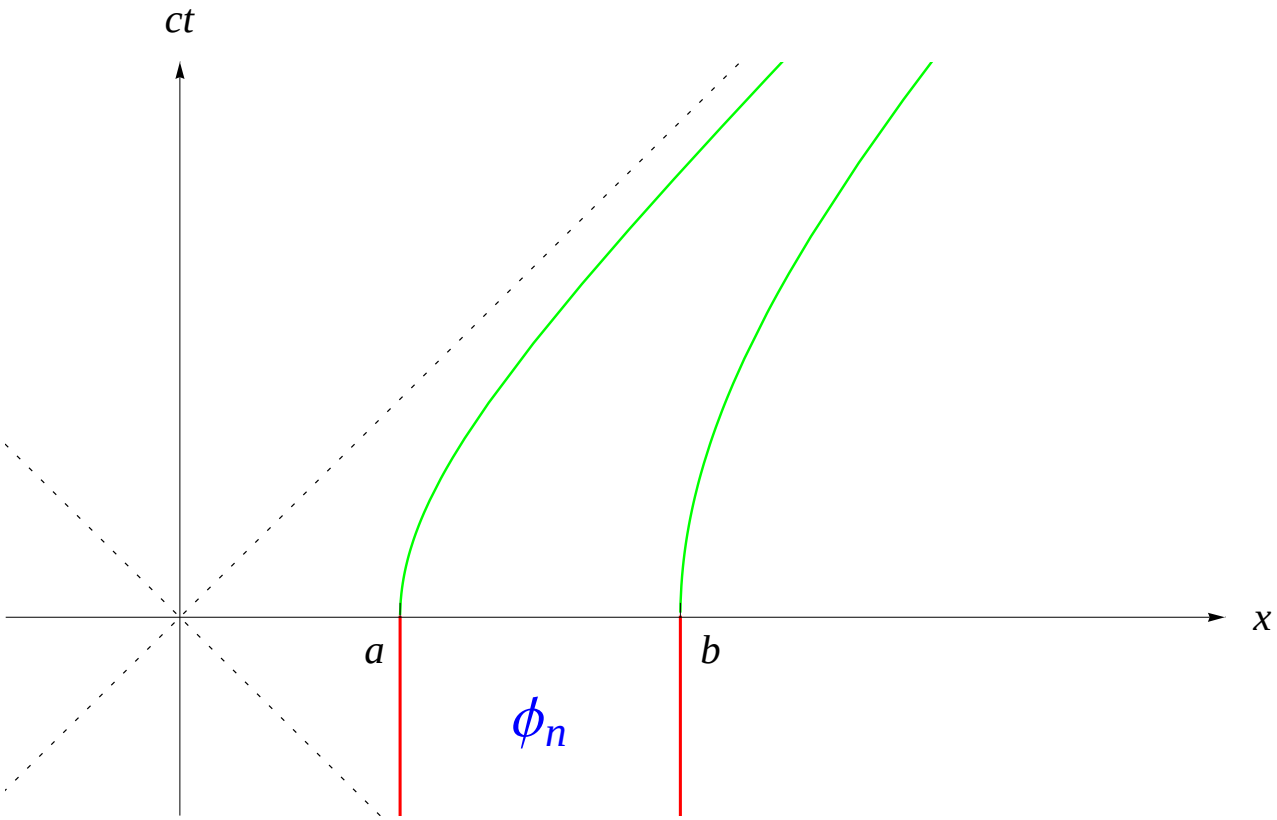
Uniform acceleration in Minkowski spacetime

Rindler coordinates (η, χ)



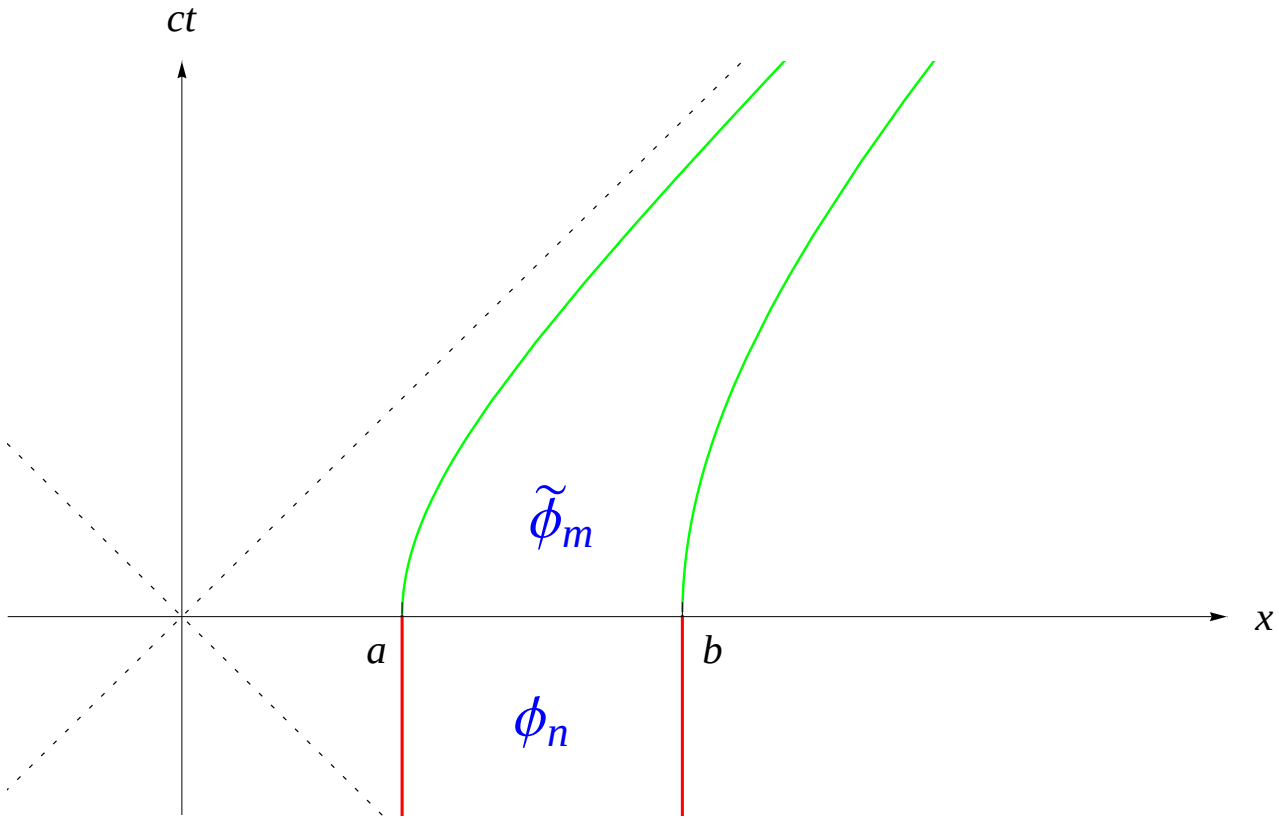
$$ct = \pm\chi \sinh\left(\frac{a\eta}{c}\right), \qquad x = \pm\chi \cosh\left(\frac{a\eta}{c}\right)$$

Quantum fields in accelerated cavities



Repeat quantisation procedure in Rindler coordinates

Quantum fields in accelerated cavities



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solutions: $\{\tilde{\phi}_n\}$: $\phi = \sum_n (\tilde{\phi}_n \, \tilde{a}_n + \tilde{\phi}_n^* \, \tilde{a}_n^\dagger)$, $[\, \tilde{a}_m, \tilde{a}_n^\dagger \,] = \delta_{mn}$

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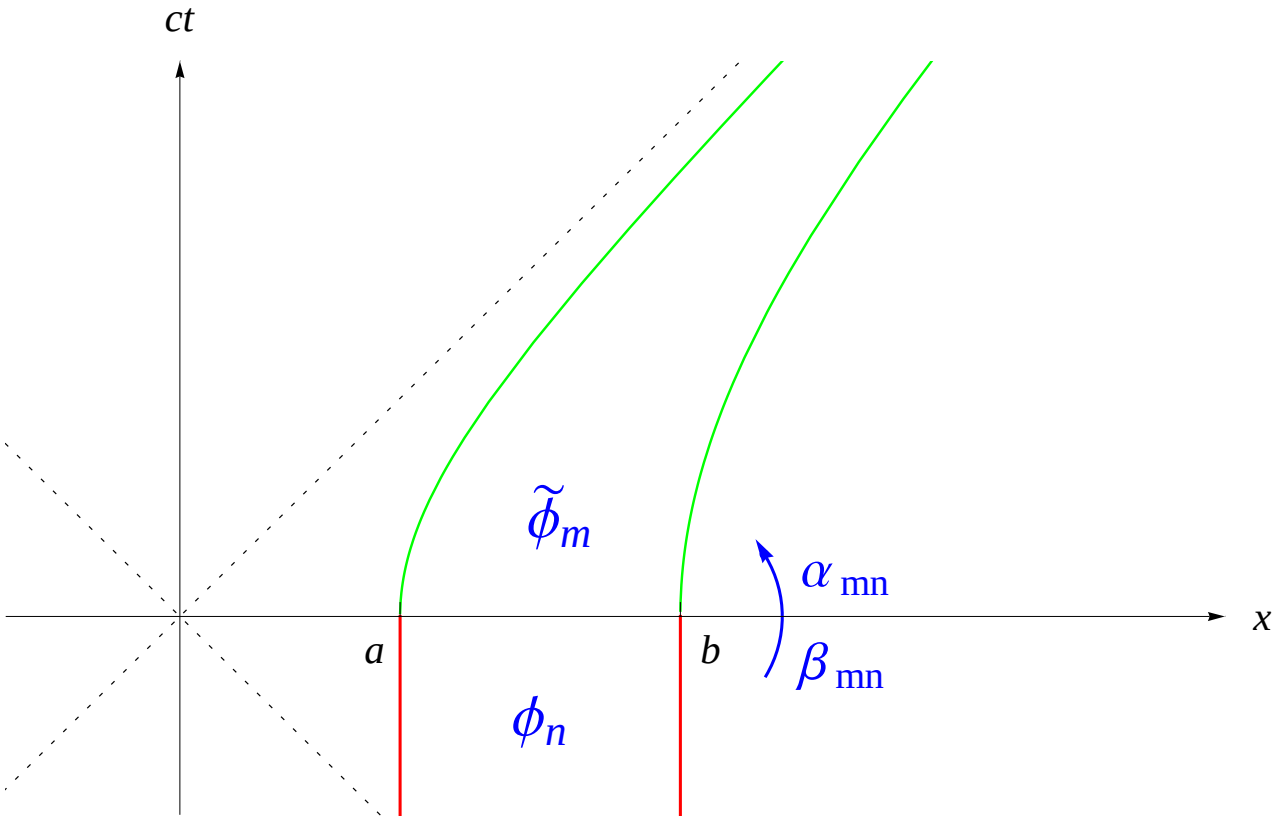

Solutions related by Bogoliubov transformation:

$$\phi_n = \sum_m (\alpha^*_{mn}\,\tilde{\phi}_m - \beta_{mn}\,\tilde{\phi}^*_m), \quad \alpha_{mn} = (\tilde{\phi}_m, \phi_n), \; \beta_{mn} = -(\tilde{\phi}_m, \phi^*_n)$$

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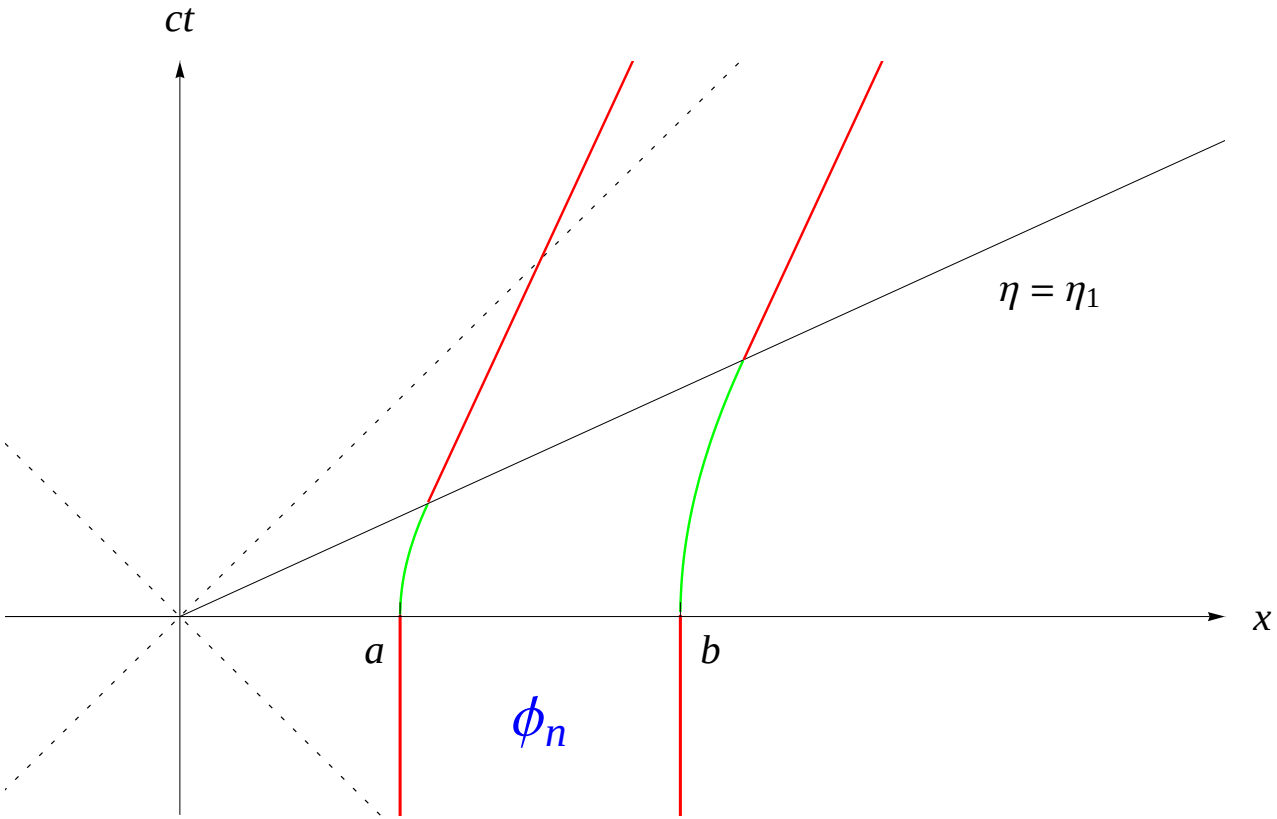
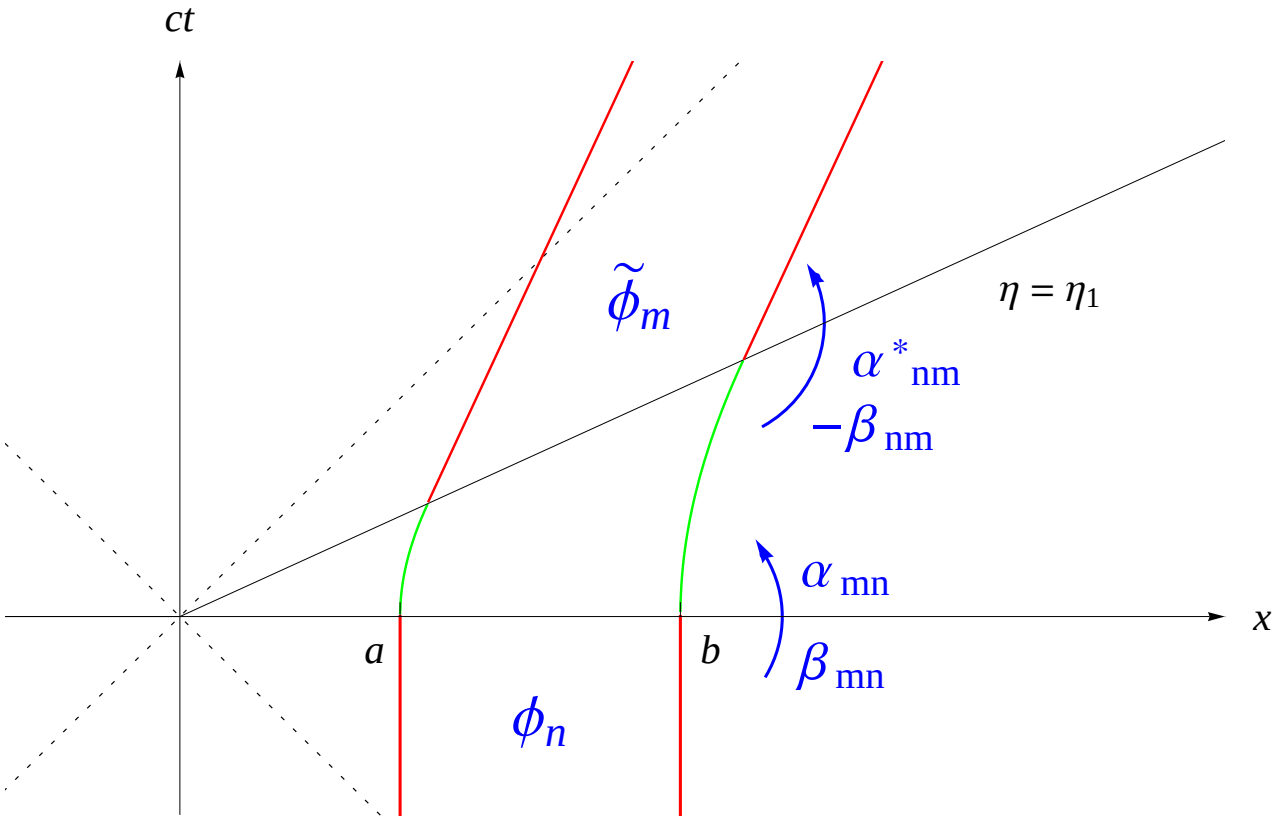


Finite duration of acceleration:

acceleration stops at fixed coordinate time $\eta = \eta_1$

Quantum fields in accelerated cavities



Finite duration of acceleration:

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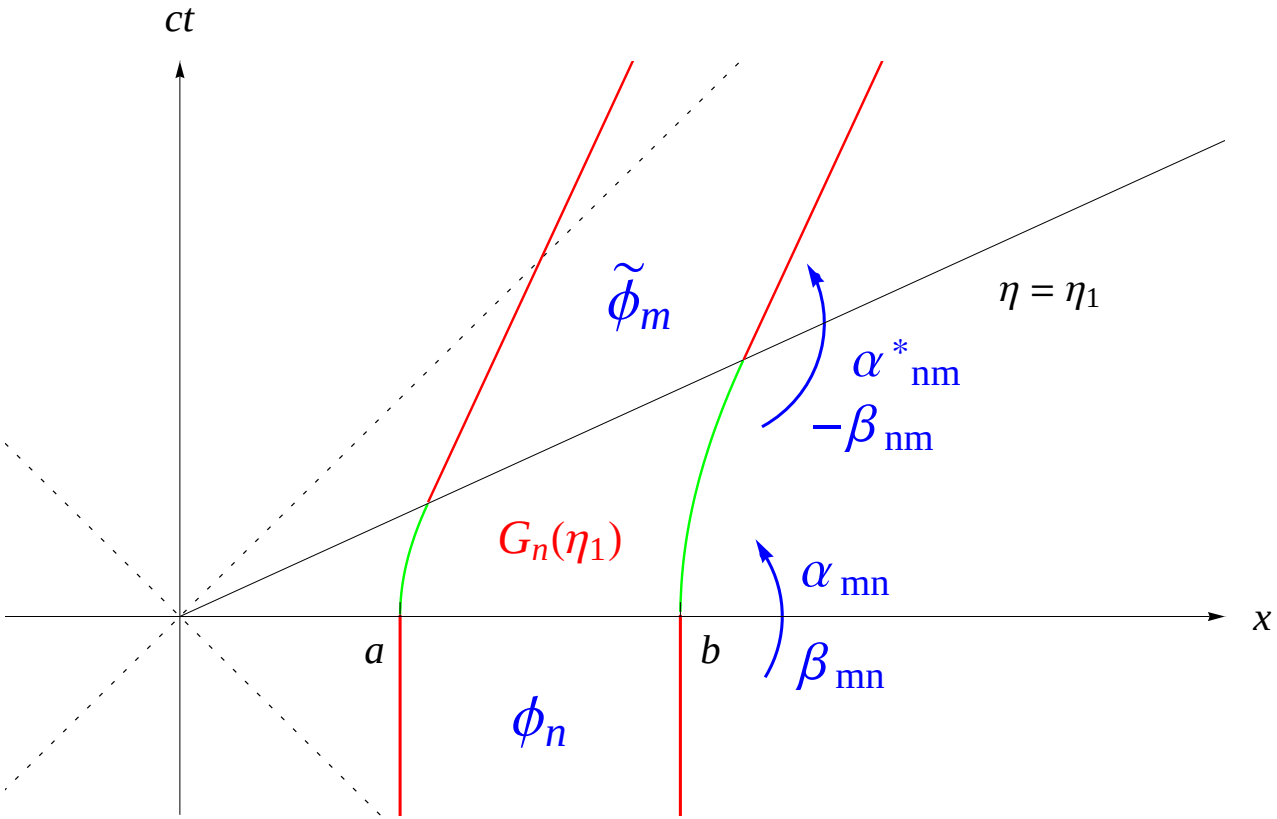

Finite duration of acceleration:

modes pick up phases $G_n(\eta_1)$

Bogoliubov Transformations

Bosons:

solutions:

$$\phi_n = \sum_m \left(\alpha^*_{mn} \tilde{\phi}_m - \beta_{mn} \tilde{\phi}^*_m \right)$$

operators:

$$a_n = \sum_m \left(\alpha_{mn} \tilde{a}_m + \beta^*_{mn} \tilde{a}^\dagger_m \right)$$

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$$\alpha_{mn} = (\tilde{\phi}_m, \phi_n), \; \beta_{mn} = -(\tilde{\phi}_m, \phi^*_n)$$

inner product:

$$(\phi_n, \phi_m) = -i \int_\Sigma \phi_n \overleftrightarrow{\partial}_t \phi^*_m \, d^{p-1}x$$

Fermions:

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Transformation of Fock states

Vacuum States

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$$|\,0\,\rangle \propto e^{W}\,|\,\tilde{0}\,\rangle$$

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$\Rightarrow$ act with creation operators on $|\,0\,\rangle$ and $\|\,0\,\rangle\rangle$

One small drawback...

have to work *perturbatively* in parameter $h := \frac{2\delta}{a+b}$

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Gaussian States - covariance matrix formalism

$$\sigma_{ij} = \langle \mathrm{X}_i \mathrm{X}_j + \mathrm{X}_j \mathrm{X}_i \rangle - 2 \langle \mathrm{X}_i \rangle \langle \mathrm{X}_j \rangle$$

quadratures: $\mathrm{X}_{(2n-1)} = \frac{1}{\sqrt{2}}(a_n + a_n^\dagger)$ and $\mathrm{X}_{(2n)} = \frac{-i}{\sqrt{2}}(a_n - a_n^\dagger)$

Bogoliubov transformation $\Rightarrow$ symplectic transformation S

$$\tilde{\sigma} = S\,\sigma\,S^T$$

$$S = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} & \cdots \\ \mathcal{M}_{21} & \mathcal{M}_{22} & \mathcal{M}_{23} & \cdots \\ \mathcal{M}_{31} & \mathcal{M}_{32} & \mathcal{M}_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \text{ where } \mathcal{M}_{mn} = \begin{pmatrix} \mathrm{Re}(\alpha_{mn} - \beta_{mn}) & \mathrm{Im}(\alpha_{mn} + \beta_{mn}) \\ -\mathrm{Im}(\alpha_{mn} - \beta_{mn}) & \mathrm{Re}(\alpha_{mn} + \beta_{mn}) \end{pmatrix}$$

Tracing becomes trivial, Entanglement measures well known

N. F. and I. Fuentes, e-print arXiv:1204.0617 [quant-ph] (2012).

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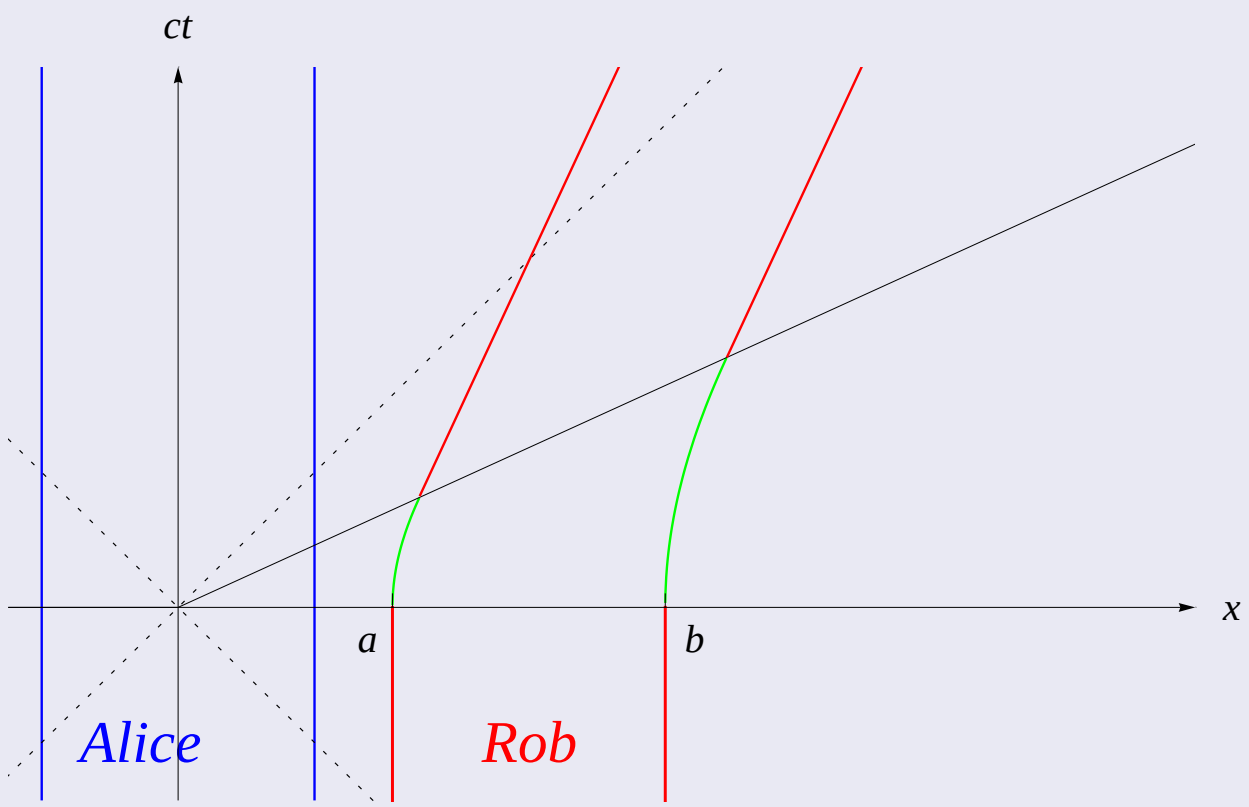
Results - Degradation of Entanglement

"In-Region": maximal entanglement (Bell state) between mode κ of inertial and mode k of accelerated cavity



"Out-Region": Trace out all other modes: degradation of entanglement

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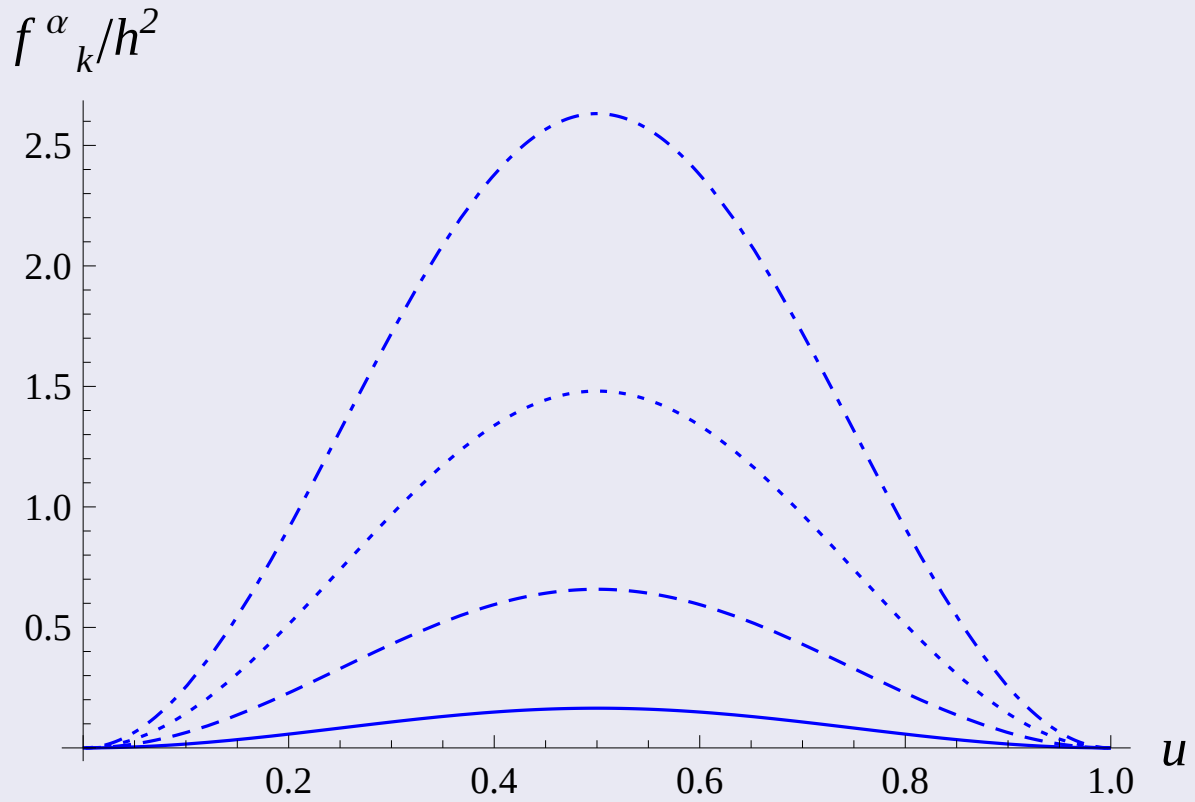

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N. F., D. E. Bruschi, J. Louko and I. Fuentes, Phys. Rev. D **85**, 081701(R) (2012).
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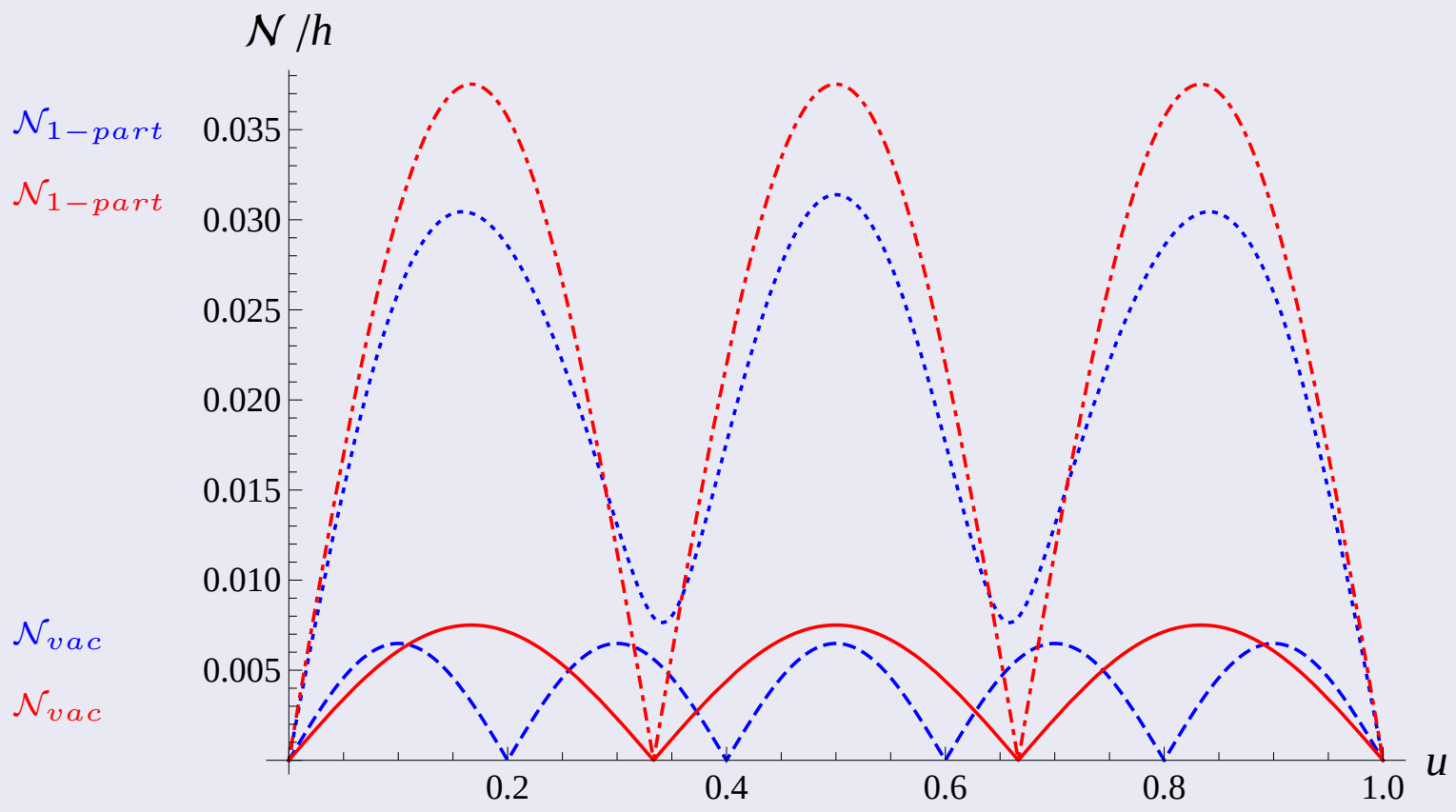
Results - Generation of Entanglement

"In-Region": separable state of modes k, k' in single cavity



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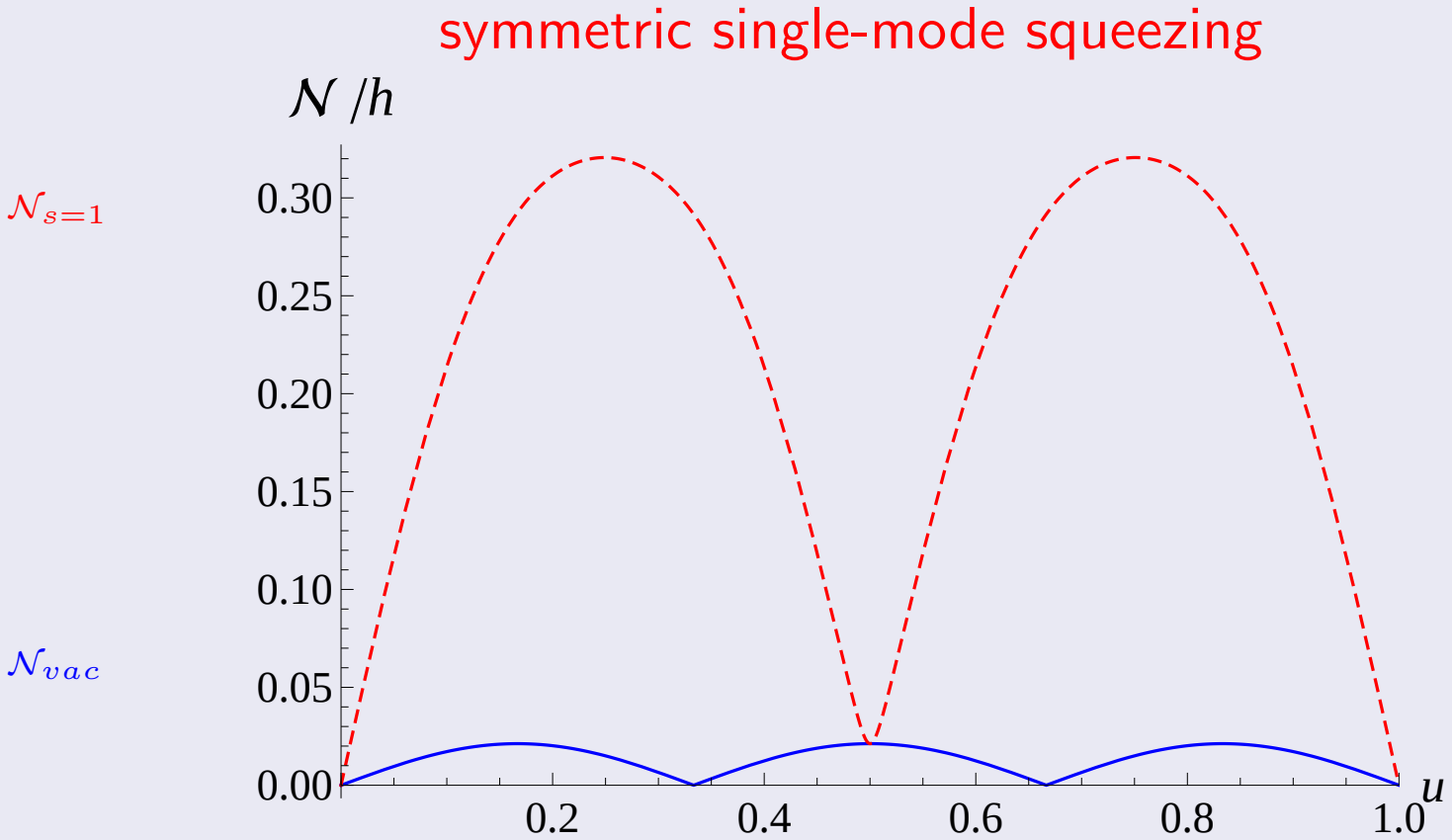

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Results - Applicability

Results obtained ...

- analytically in terms of abstract coefficients
- perturbatively (to second order) in h
- for arbitrary uncharged scalar and charged Dirac fields
- for a generic class of cavity trajectories
- are periodic in time for $(1+1)$-dim massless fields
- formalism applicable in curved backgrounds and for Gaussian states![1]
- effects enhanced by mass, initial squeezing and resonances![2]

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Thank you for your attention