

Residual entanglement of accelerated fermions is not nonlocal

RQI-N Madrid, September 6th, 2011

Nicolai Friis

The University of Nottingham

in collaboration with

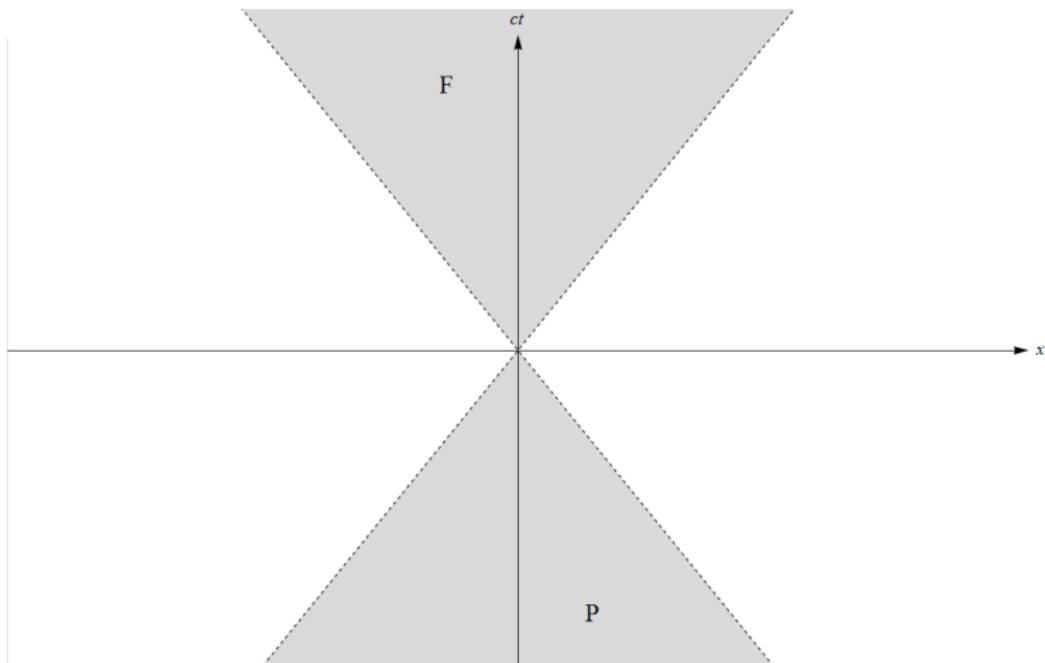
Philipp Köhler¹, Eduardo Martín-Martínez² and Reinhold A. Bertlmann¹

¹ Faculty of Physics, University of Vienna

² Instituto de Física Fundamental, CSIC

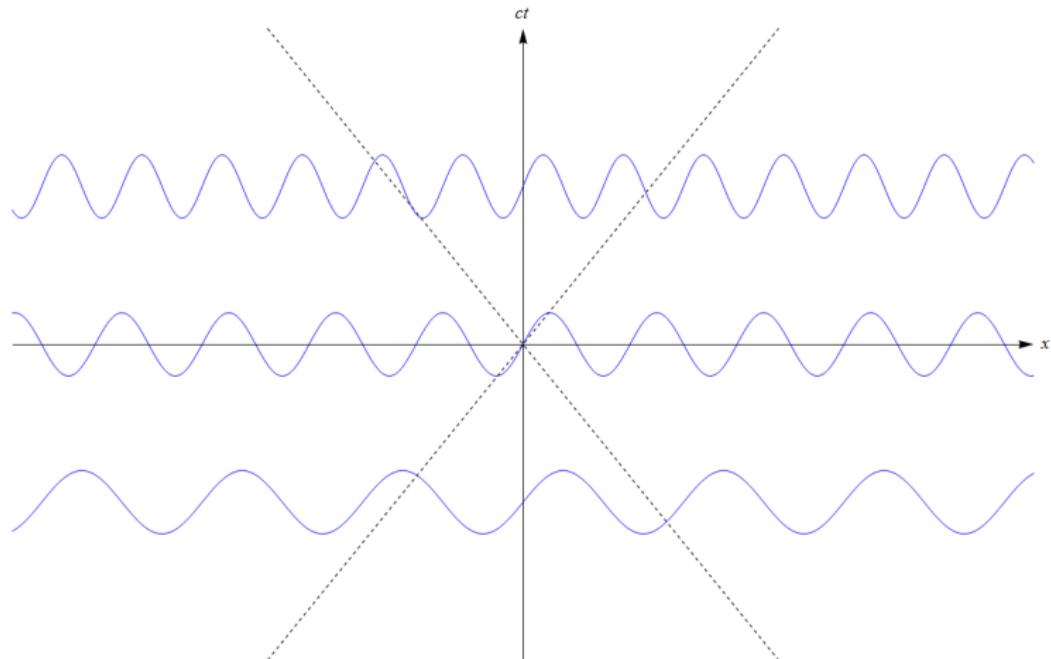
Acceleration & Entanglement

Minkowski space



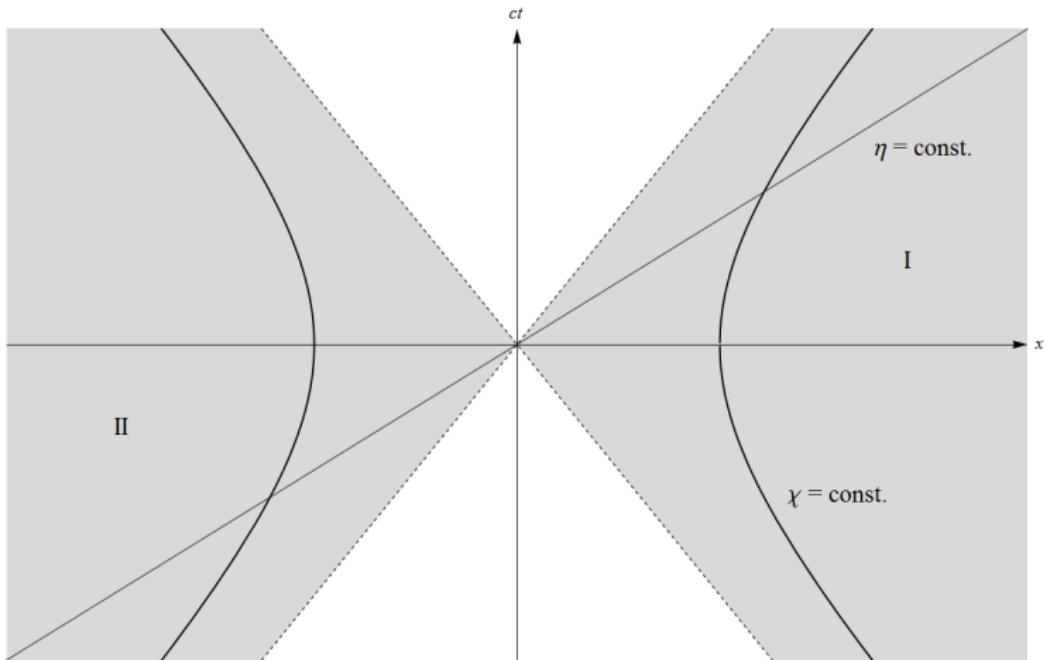
Inertial observers \Rightarrow straight lines from P to F

Minkowski modes



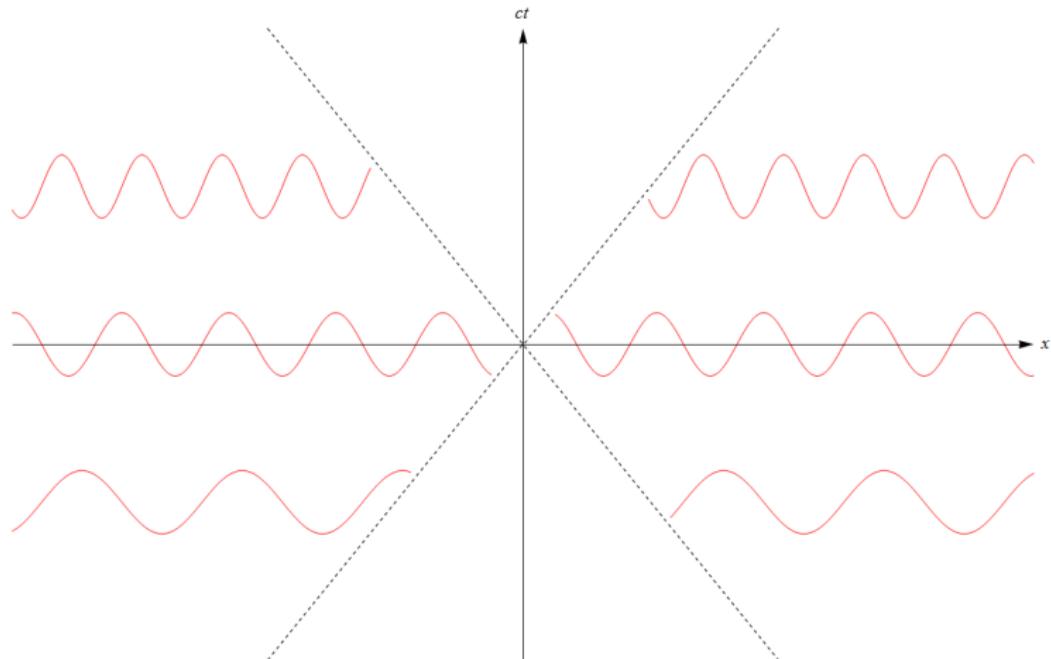
quantum field ϕ expanded in Minkowski modes

Rindler coordinates (η, χ)



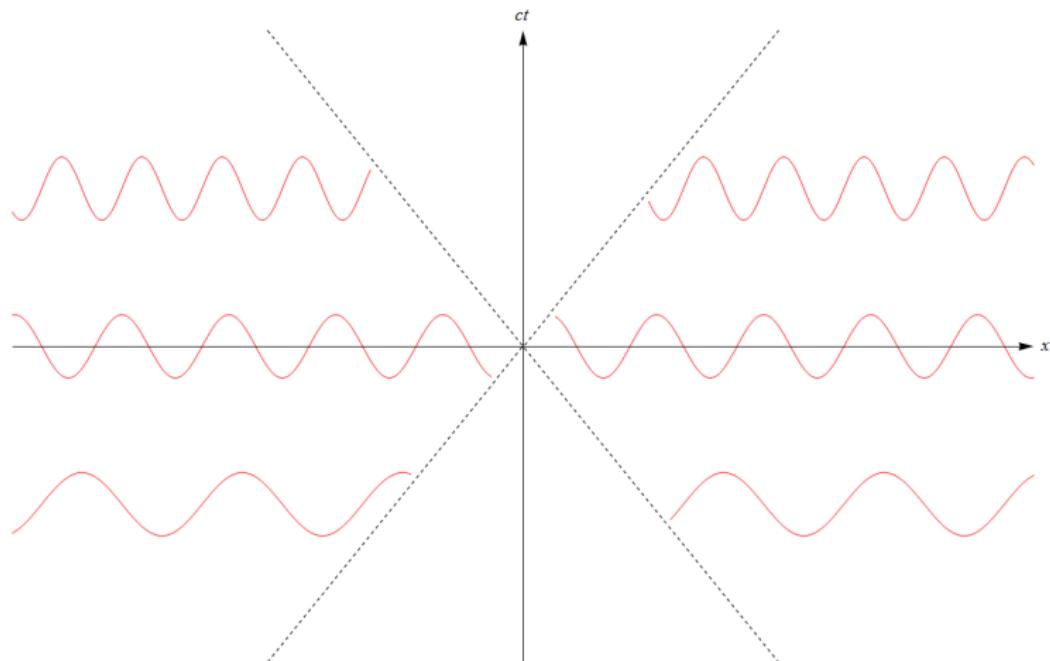
$$ct = \pm \chi \sinh \left(\frac{a\eta}{c} \right), \quad x = \pm \chi \cosh \left(\frac{a\eta}{c} \right)$$

Rindler modes



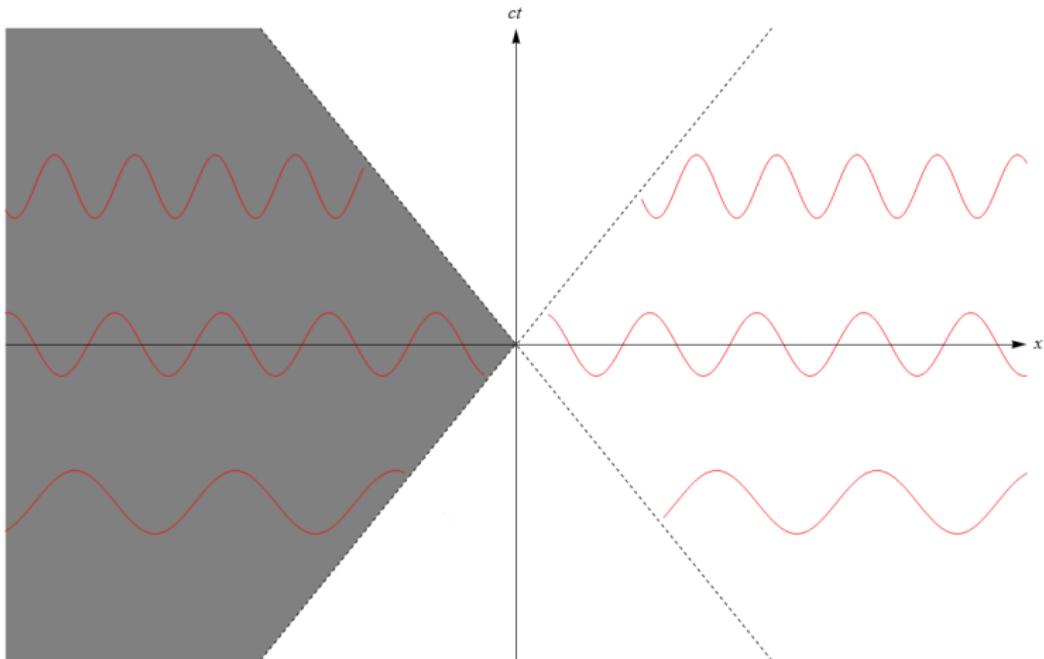
quantum field ϕ expanded in **Rindler modes**

Unruh effect



regions I and II causally disconnected

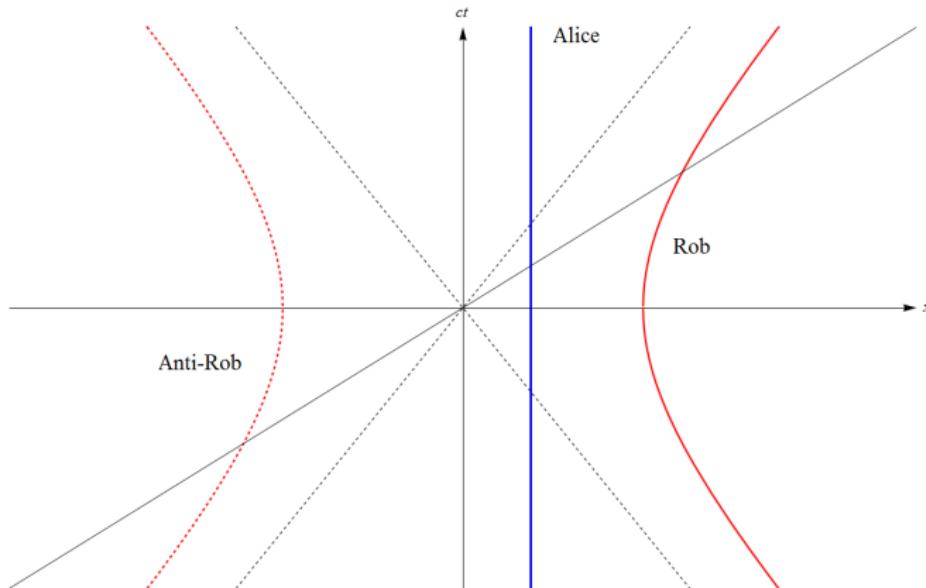
Thermalization



\Rightarrow tracing over region I or II \Rightarrow thermalization

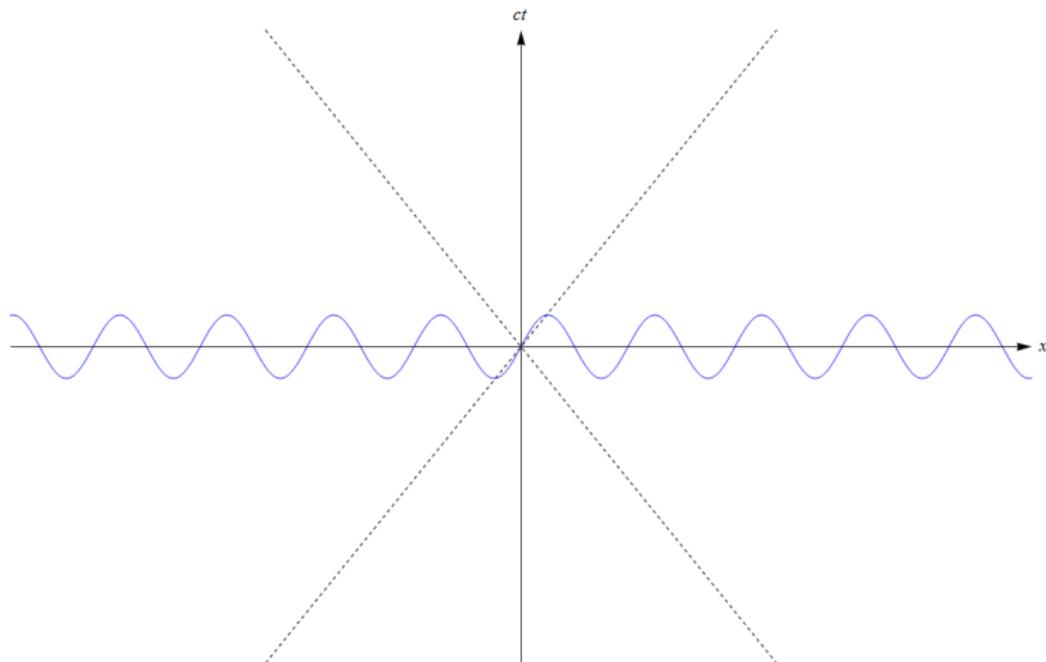
Accelerated & inertial observers share entangled state

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|0_\omega\rangle_A |0_\Omega\rangle_U + |1_\omega\rangle_A |1_\Omega\rangle_U)$$



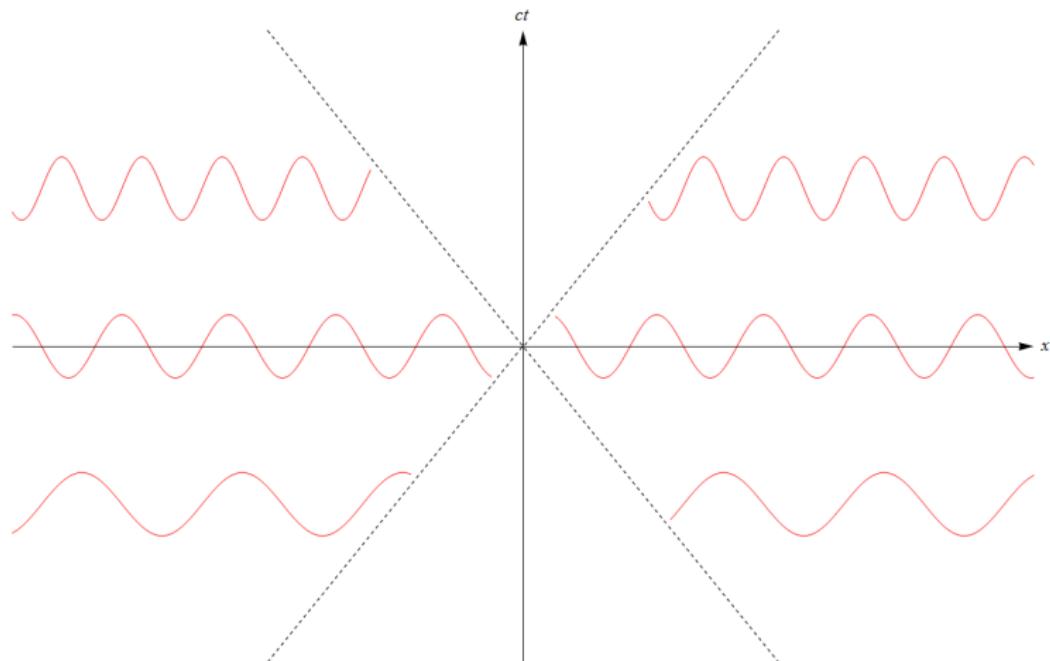
each observer has access to a particular mode of the field

Transformation of modes



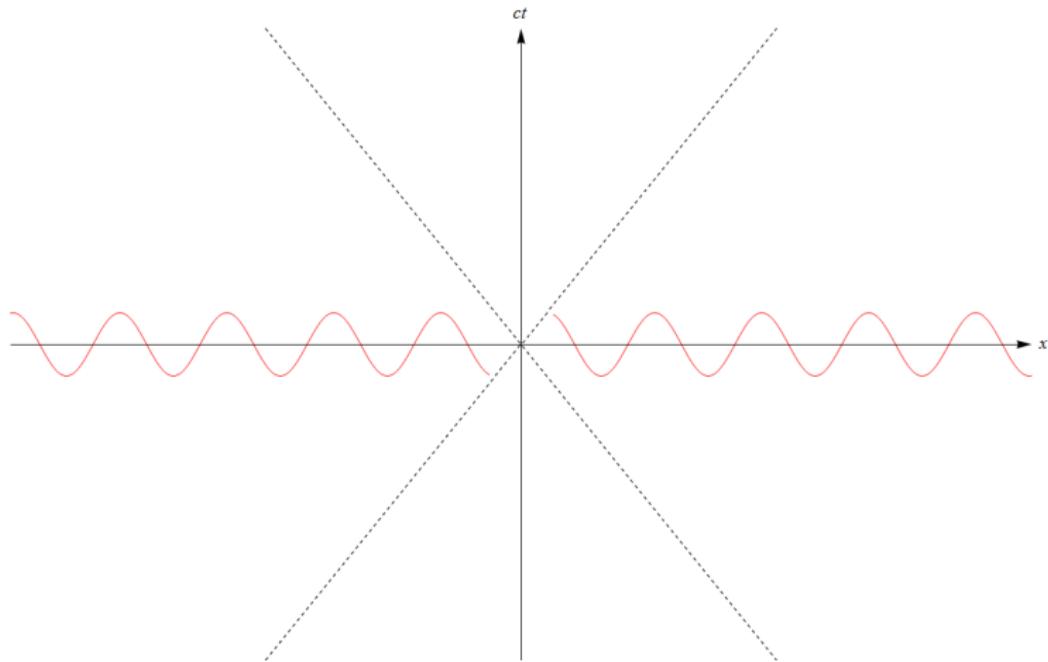
Minkowski mode transformed to ...

Transformation of modes



Minkowski mode transformed to ... multiple Rindler modes

Single mode approximation



Minkowski mode transformed to ... single Rindler mode

Unruh modes

Alternatively: Unruh mode instead of Minkowski mode

- ▶ one Unruh frequency transforms to one Rindler frequency
- ▶ two independent types of Unruh modes: L,R

$$C_{\Omega,R/L} \equiv \left(\cos r_\Omega c_{\Omega,I/\text{II}} - \sin r_\Omega d_{\Omega,\text{II}/I}^\dagger \right)$$

where $c_{\Omega,I/\text{II}}$, $d_{\Omega,\text{II}/I}^\dagger$ annihilate and create particles/antiparticles in Rindler regions I and II respectively, and $\tan r_\Omega = e^{-\pi\Omega/a}$

- ▶ general Unruh mode: superposition of L,R modes

$$c_{\Omega,U}^\dagger = q_L (C_{\Omega,L}^\dagger \otimes \mathbb{1}_R) + q_R (\mathbb{1}_L \otimes C_{\Omega,R}^\dagger)$$

D. E. Bruschi, J. Louko, E. Martín-Martínez, A. Dragan and I. Fuentes, Phys. Rev. A **82**, 042332 (2010)

Tracing \Rightarrow loss of information/decoherence

Bosons¹

- ▶ entanglement **decreases** with acceleration
- ▶ entanglement **vanishes** in infinite acceleration limit

Fermions²

- ▶ entanglement **decreased** w.r.t. to inertial observers
- ▶ entanglement **swapped**: $\text{Rob} \Leftrightarrow \text{Anti-Rob}$
 $\text{particles} \Leftrightarrow \text{antiparticles}$
- ▶ Generally **nonvanishing** in infinite acceleration limit

¹I. Fuentes-Schuller and R. B. Mann, Phys. Rev. Lett. **95**, 120404 (2005)

²P. M. Alsing, I. Fuentes-Schuller, R. B. Mann and T. E. Tessier, Phys. Rev. A **74**, 032326 (2006); E. Martín-Martínez and I. Fuentes, Phys. Rev. A **83**, 052306 (2011)

Transformation of states

Initial state

$$|\psi_+\rangle = \frac{1}{\sqrt{2}} (|0_\omega\rangle_A |0_\Omega\rangle_U + |1_\omega\rangle_A |1_\Omega\rangle_U^+)$$

(Anti-)Rob's states \Downarrow \Downarrow

transformed to $\sum_{i,j,k,l} c_{ijkl} |i_\Omega\rangle_I^+ |j_\Omega\rangle_I^- |k_\Omega\rangle_{II}^+ |l_\Omega\rangle_{II}^-$

Tracing

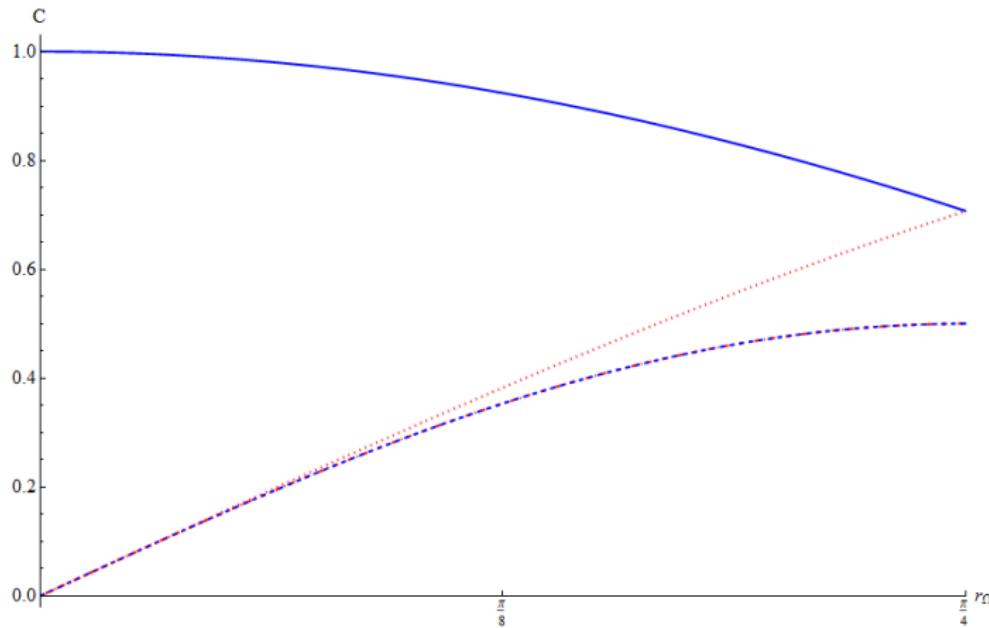
tracing over regions I (Rob) or II (Anti-Rob)

and either particle or antiparticle sector

\Rightarrow reduced states: ρ_{PAR+} , $\rho_{A\bar{R}+}$, ρ_{PAR-} , $\rho_{A\bar{R}-}$

Fermion entanglement

$q_R = 1.00$

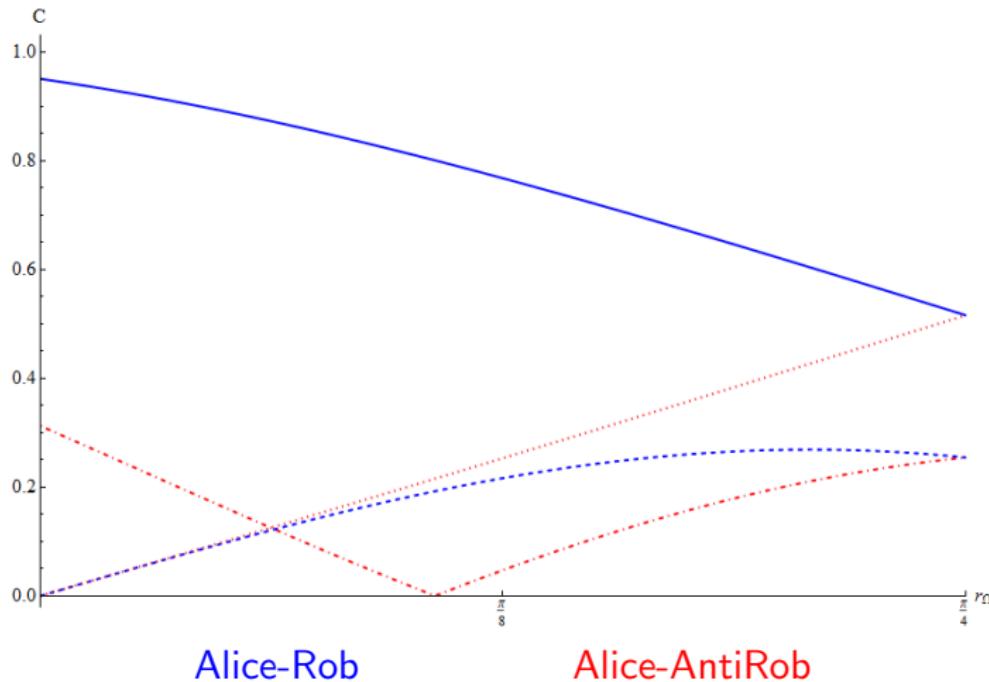


Alice-Rob $\rho_{AR\pm}$

Alice-AntiRob $\rho_{A\bar{R}\pm}$

Fermion entanglement

$$q_R = 0.95$$

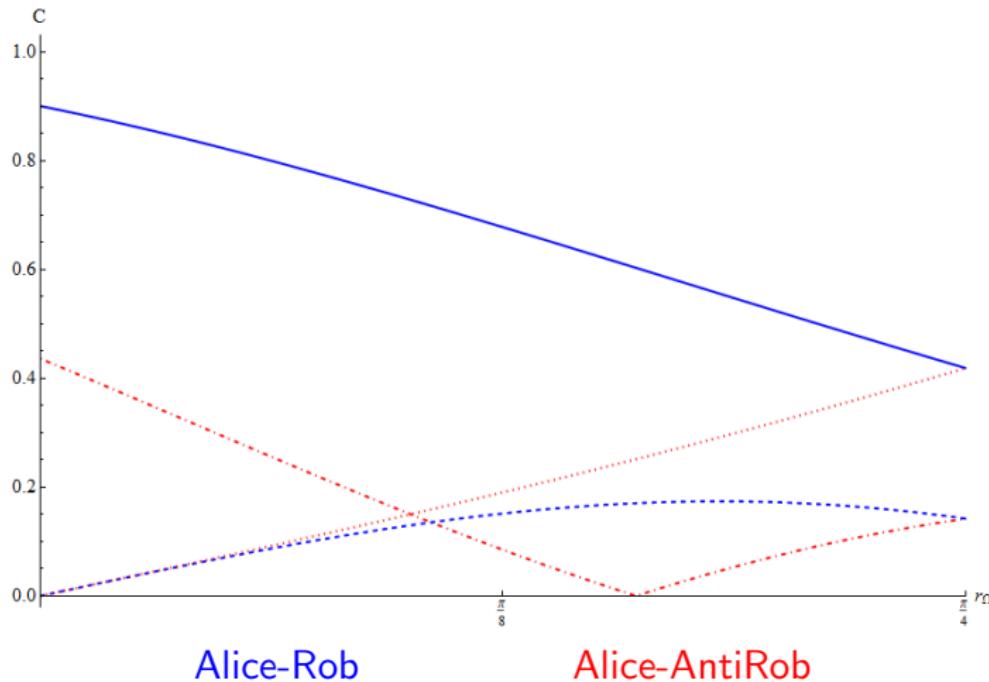


Alice-Rob

Alice-AntiRob

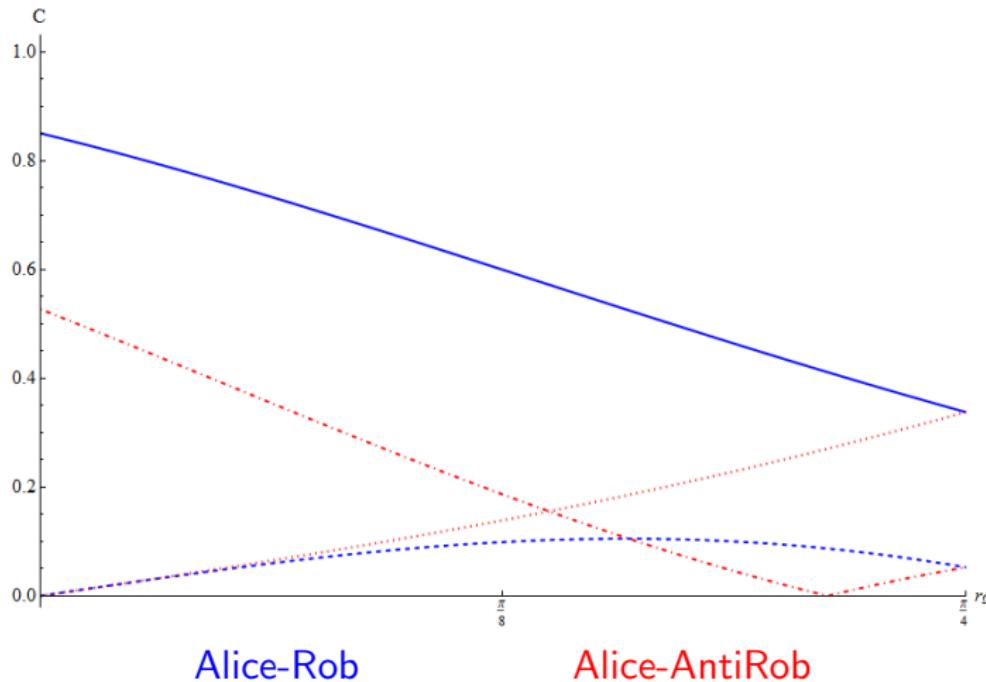
Fermion entanglement

$$q_R = 0.90$$



Fermion entanglement

$$q_R = 0.85$$

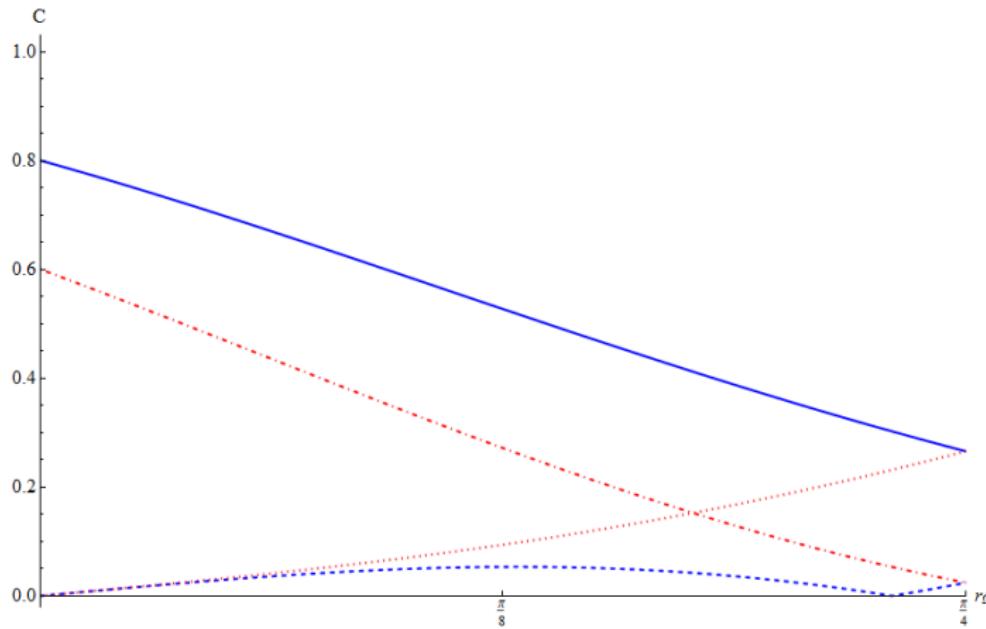


Alice-Rob

Alice-AntiRob

Fermion entanglement

$$q_R = 0.80$$

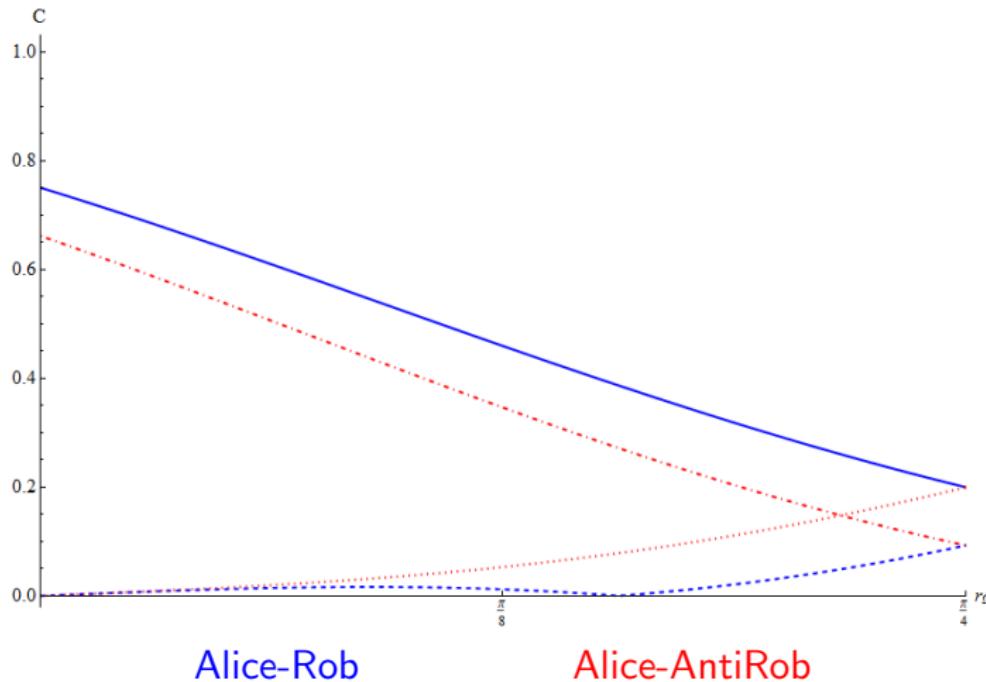


Alice-Rob

Alice-AntiRob

Fermion entanglement

$$q_R = 0.75$$

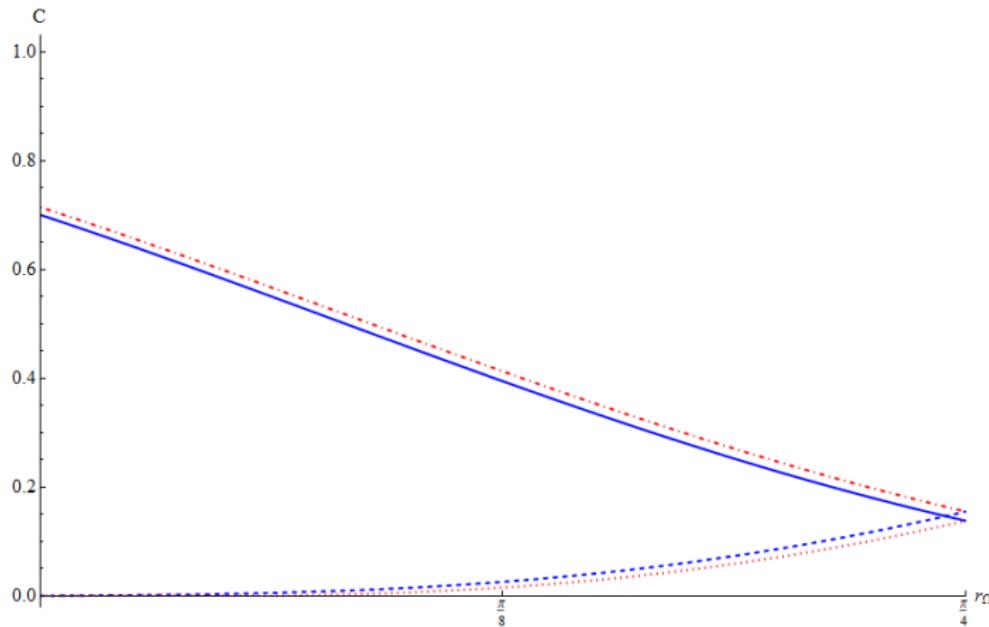


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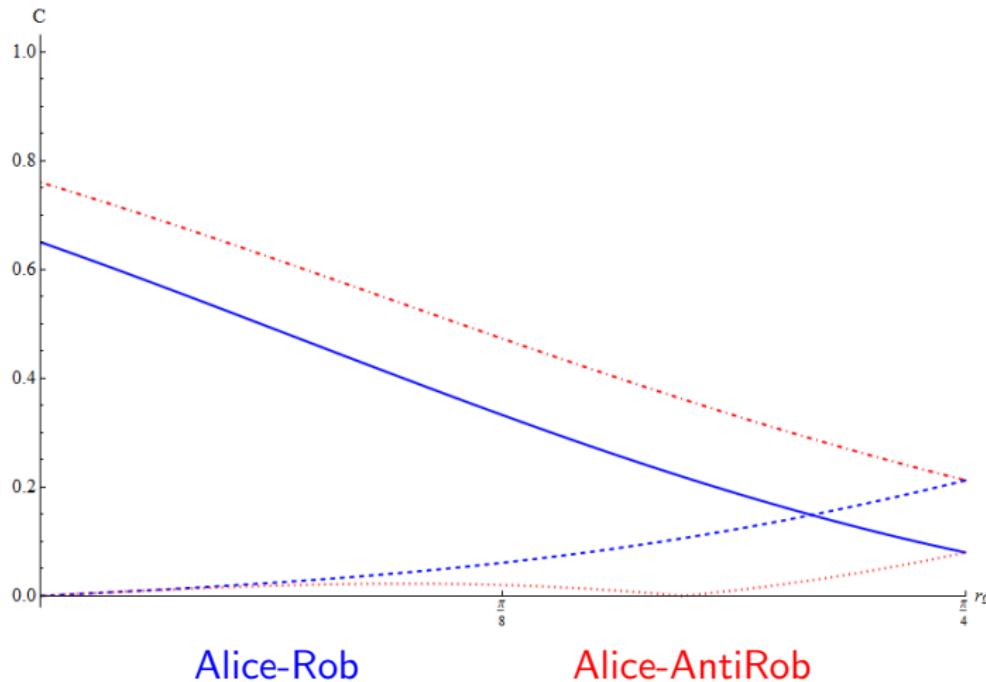


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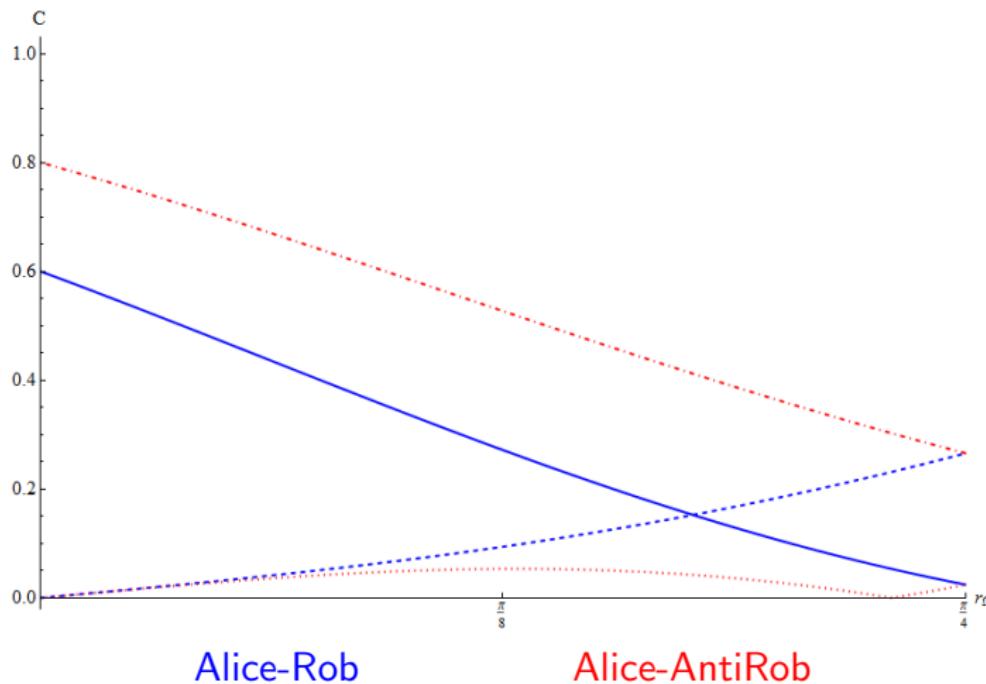


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Alice-AntiRob

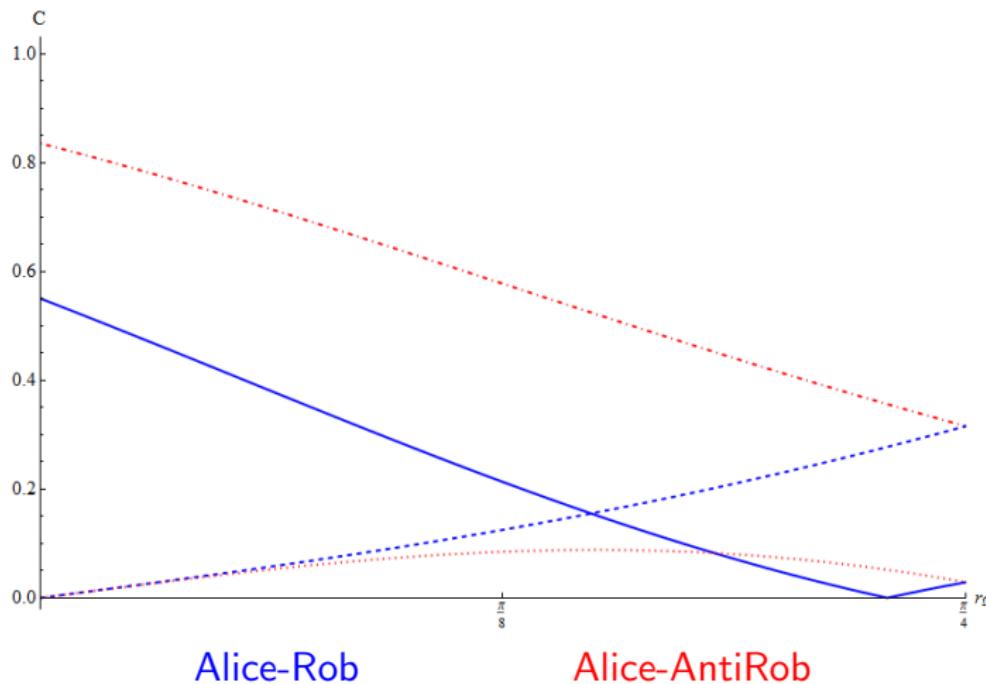
Fermion entanglement

$$q_R = 0.60$$



Fermion entanglement

$$q_R = 0.55$$

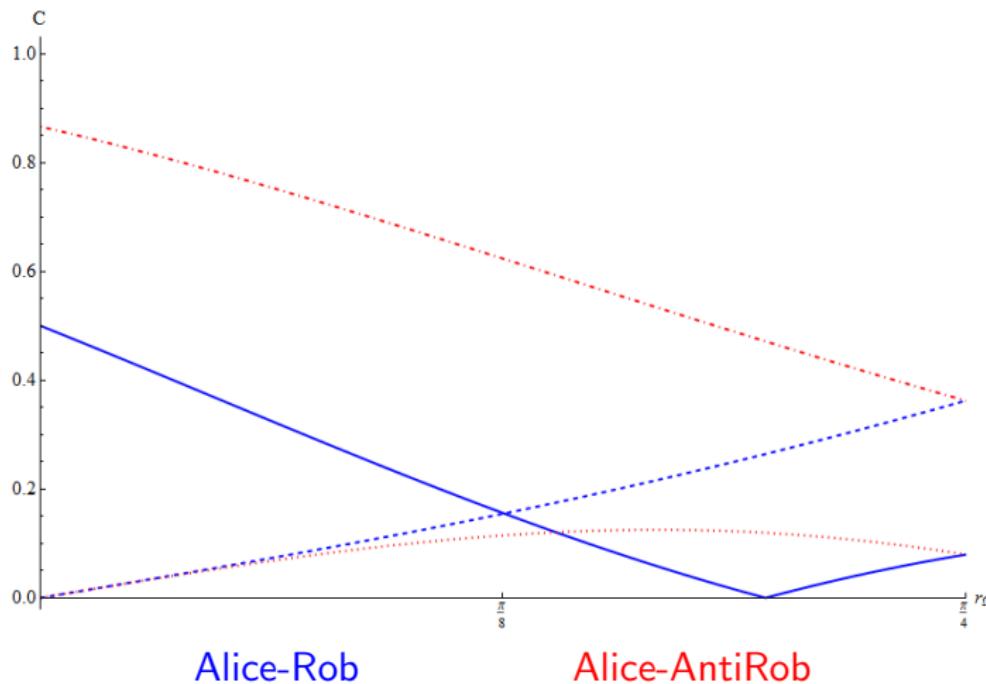


Alice-Rob

Alice-AntiRob

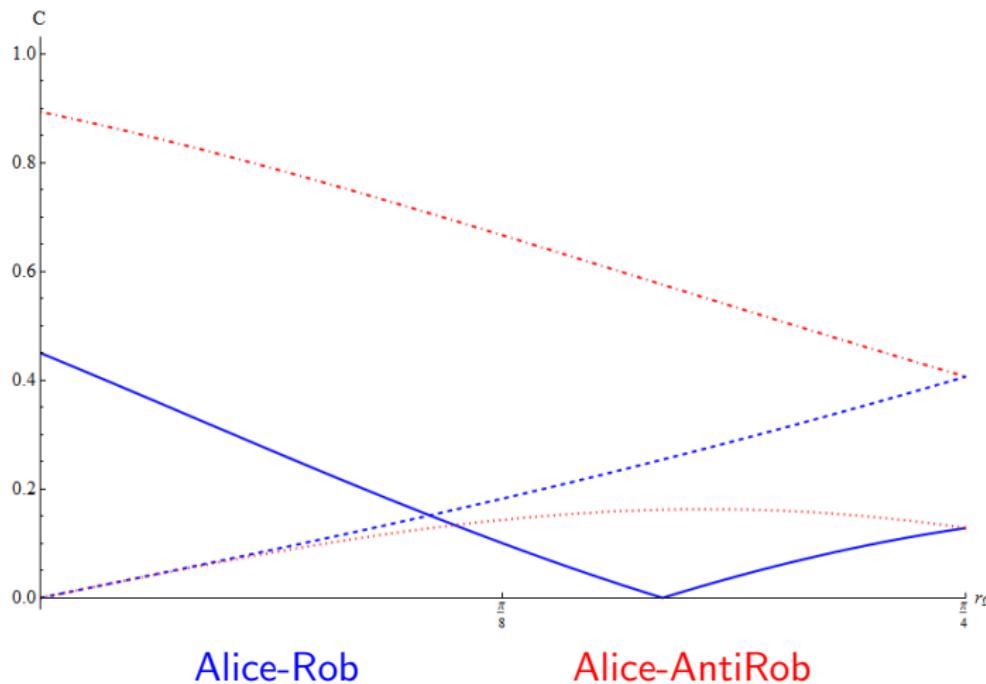
Fermion entanglement

$q_R = 0.50$



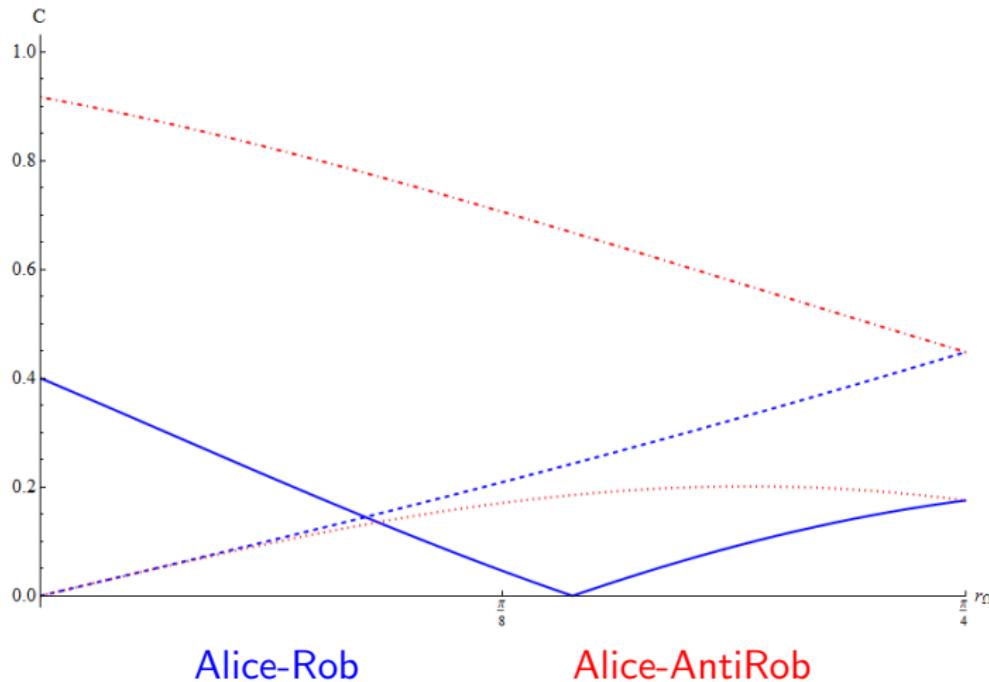
Fermion entanglement

$$q_R = 0.45$$



Fermion entanglement

$$q_R = 0.40$$

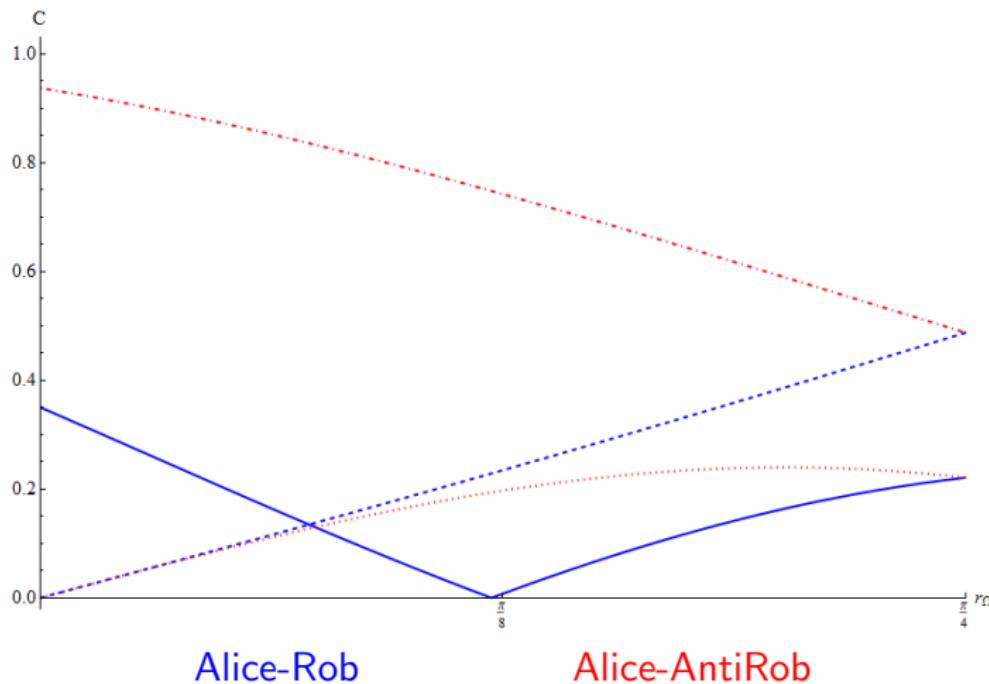


Alice-Rob

Alice-AntiRob

Fermion entanglement

$$q_R = 0.35$$

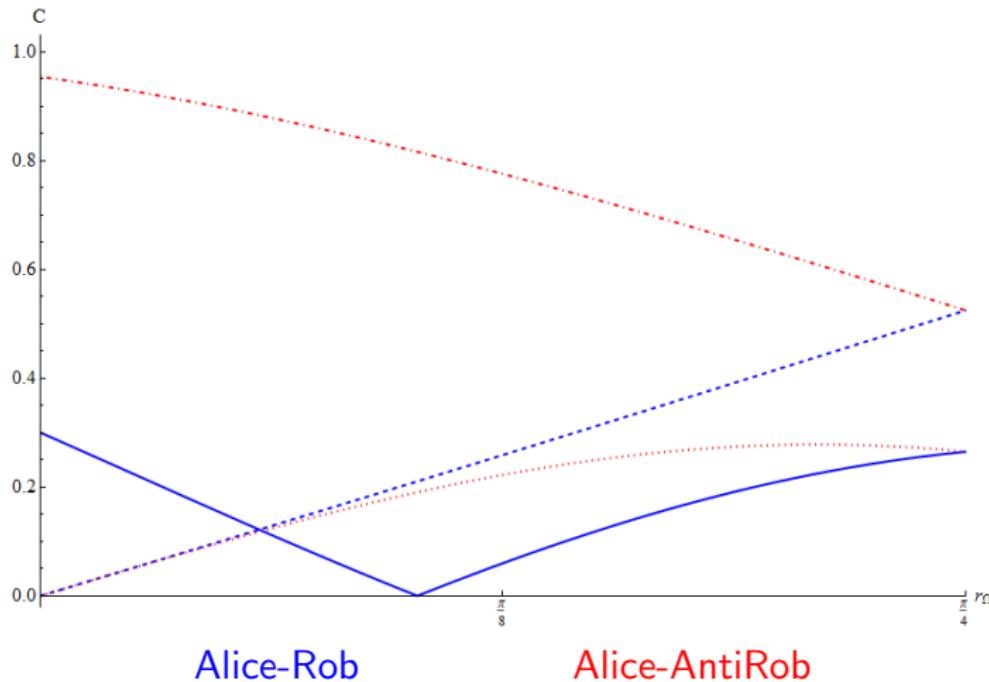


Alice-Rob

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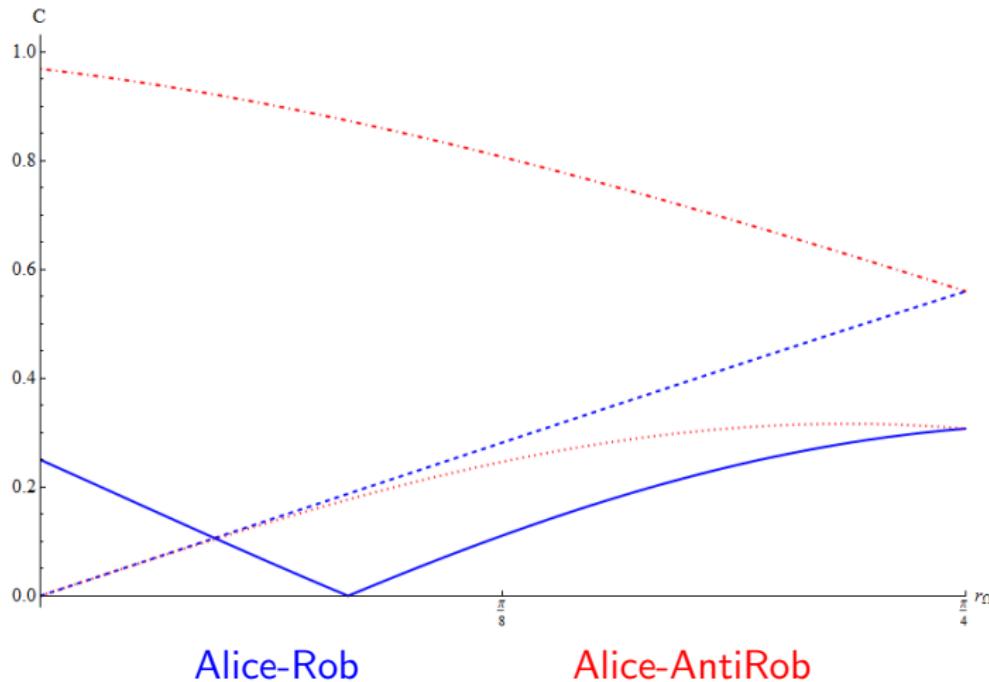
Fermion entanglement

$$q_R = 0.30$$



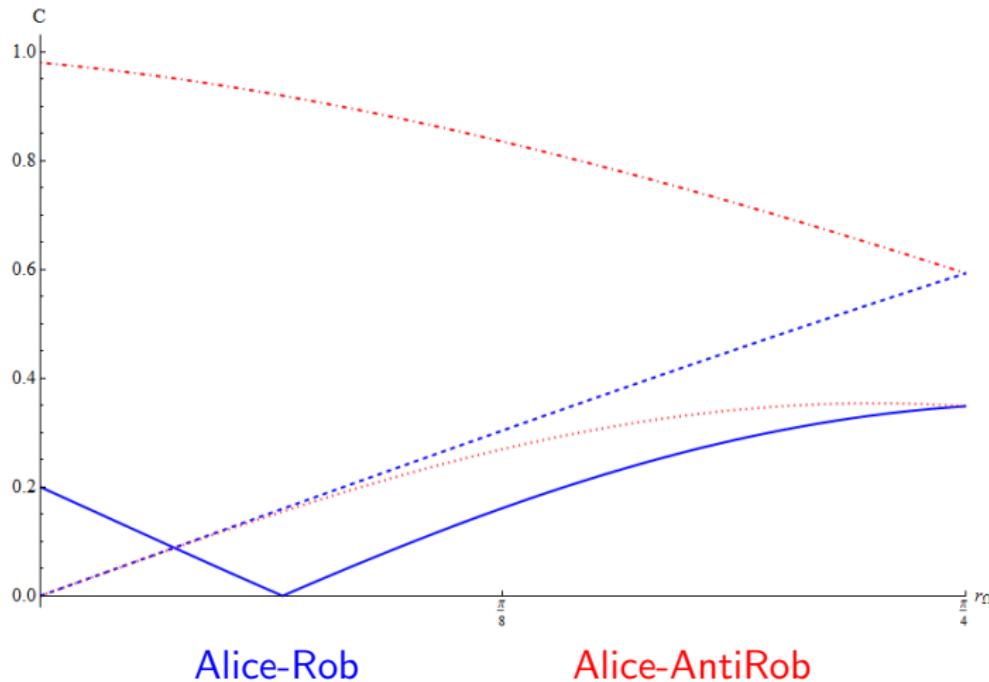
Fermion entanglement

$q_R = 0.25$



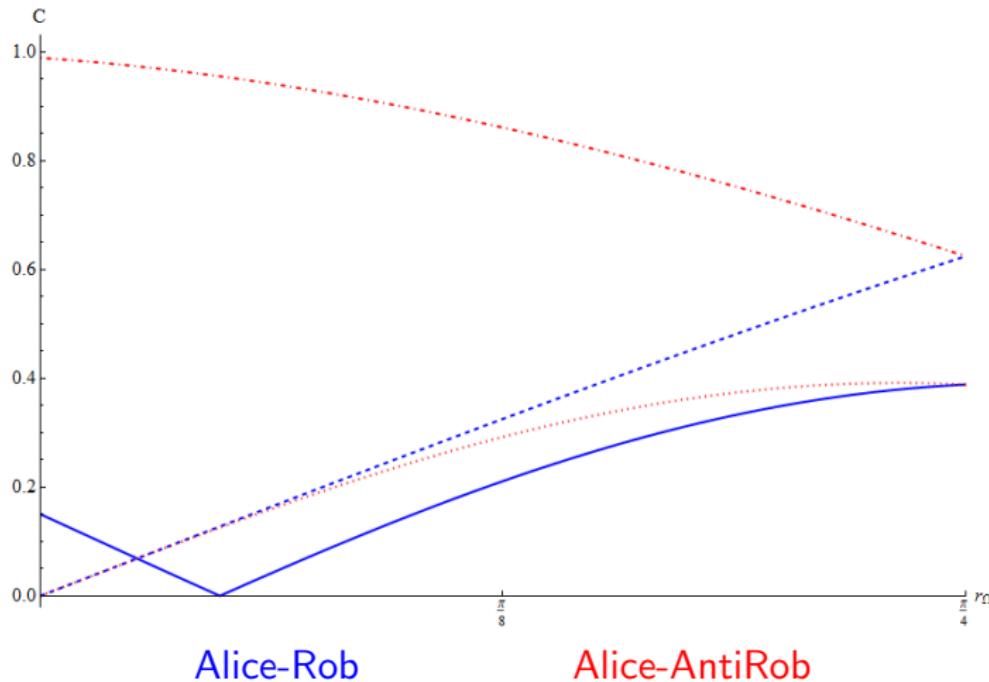
Fermion entanglement

$q_R = 0.20$



Fermion entanglement

$q_R = 0.15$

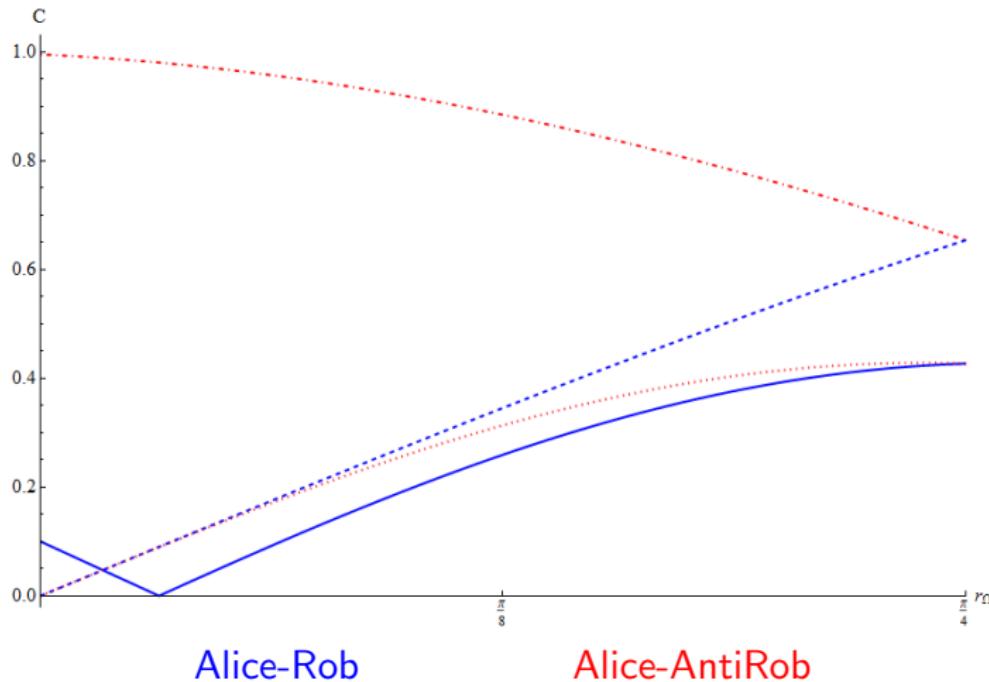


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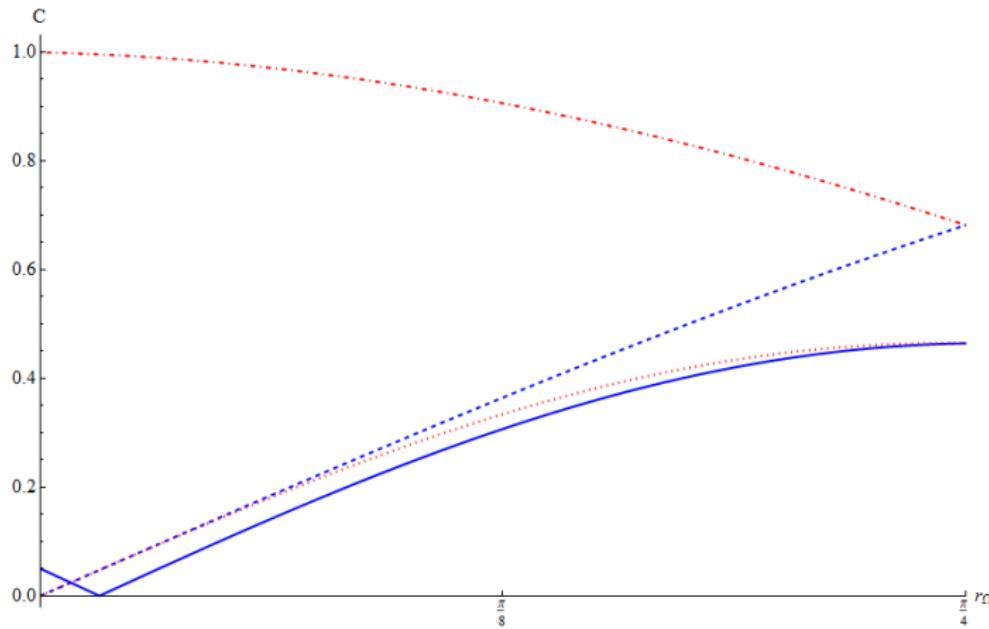
Fermion entanglement

$q_R = 0.10$



Fermion entanglement

$q_R = 0.05$

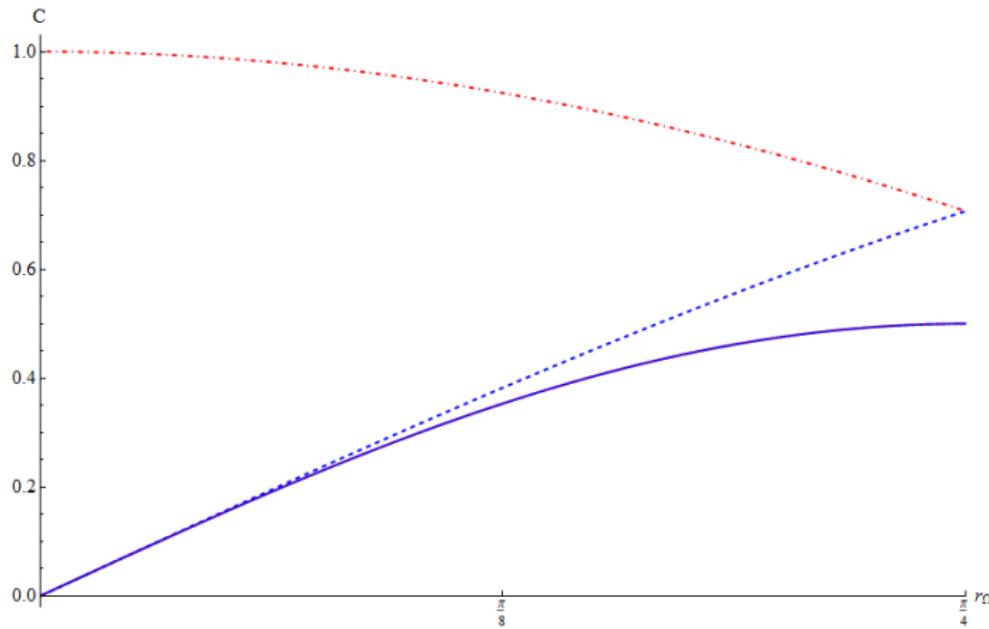


Alice-Rob

Alice-AntiRob

Fermion entanglement

$q_R = 0.00$



Alice-Rob

Alice-AntiRob

Nonlocality of Fermions

Nonlocality \Leftrightarrow Bell inequality violation

$$\text{CHSH inequality: } |\langle \mathcal{B}_{CHSH} \rangle_{\rho}| \leq 2$$

where $\mathcal{B}_{CHSH} = \mathbf{a} \cdot \sigma \otimes (\mathbf{b} + \mathbf{b}') \cdot \sigma + \mathbf{a}' \cdot \sigma \otimes (\mathbf{b} - \mathbf{b}') \cdot \sigma$
 $\mathbf{a}, \mathbf{a}', \mathbf{b}$ and $\mathbf{b}' \dots$ unit vectors in \mathbb{R}^3

- ▶ cannot be violated by local realistic theories
- ▶ can be violated by QM: maximal value $2\sqrt{2}$
- ▶ given a state: what is maximally possible violation?
 \Rightarrow maximization over $\mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}'$

CHSH criterion

Bloch representation

$$\rho = \frac{1}{4} (\mathbb{1}_2 \otimes \mathbb{1}_2 + \mathbf{r}_i \sigma_i \otimes \mathbb{1}_2 + \mathbb{1}_2 \otimes \mathbf{s}_j \sigma_j + \mathbf{t}_{mn} \sigma_m \otimes \sigma_n)$$

correlation matrix $T_\rho = (t_{mn})$ determines $\langle \mathcal{B}_{CHSH} \rangle_\rho$

CHSH criterion³

maximally possible violation of CHSH inequality for state ρ

$$\langle \mathcal{B}_{max} \rangle_\rho = 2 \sqrt{\mu_1 + \mu_2}$$

where μ_1, μ_2 are the two largest eigenvalues of $U(\rho) = T_\rho^T T_\rho$

³R., P. and M. Horodecki, Phys. Lett. A 200 340-344 (1995)

Nonlocality of Fermions

Analytical results for possible CHSH violation⁴

- ▶ Alice-Rob (particle sector): $\langle \mathcal{B}_{max} \rangle_{\rho_{AR+}} = 2\sqrt{2} |q_R| \cos r_\Omega$
- ▶ Alice-Rob (antiparticle sector): $\langle \mathcal{B}_{max} \rangle_{\rho_{AR-}^+} = 2\sqrt{2} |q_L| \sin r_\Omega$
- ▶ Alice-AntiRob: exchange q_R and q_L

Exclusive non-locality

CHSH inequality can only ever be violated by

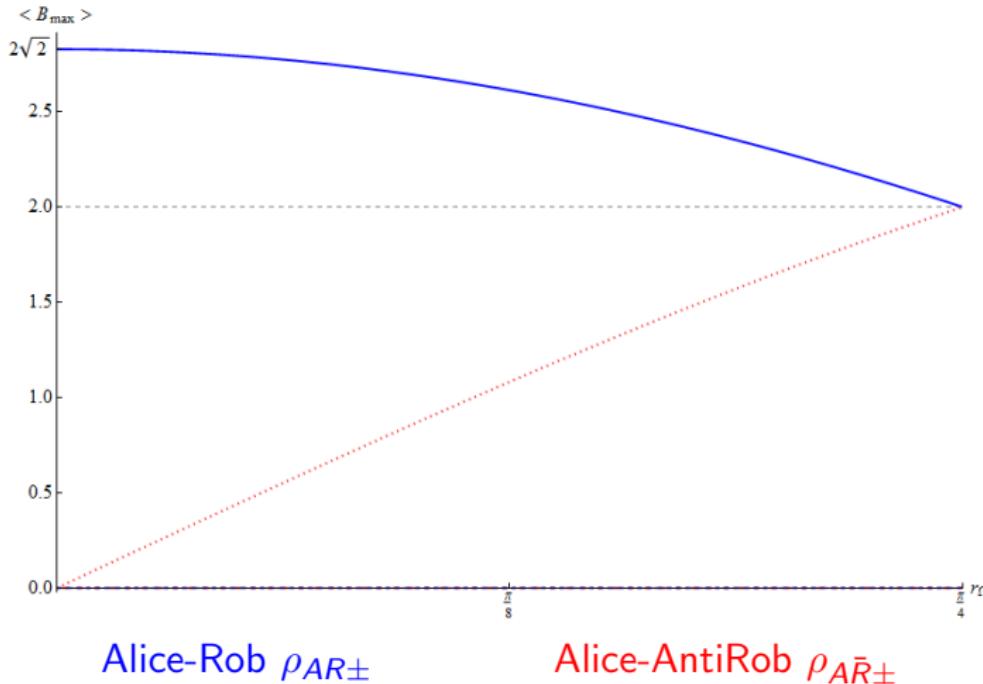
either Alice-Rob or Alice-AntiRob

$$(\langle \mathcal{B}_{max} \rangle_{\rho_{AR+}^+} + \langle \mathcal{B}_{max} \rangle_{\rho_{A\bar{R}+}^+})/2 \leq 2$$

⁴N. F., P. Köhler, E. Martín-Martínez, R. A. Bertlmann, arXiv:1107.3235v1
A. Smith and R. B. Mann, arXiv:1107.4633v1

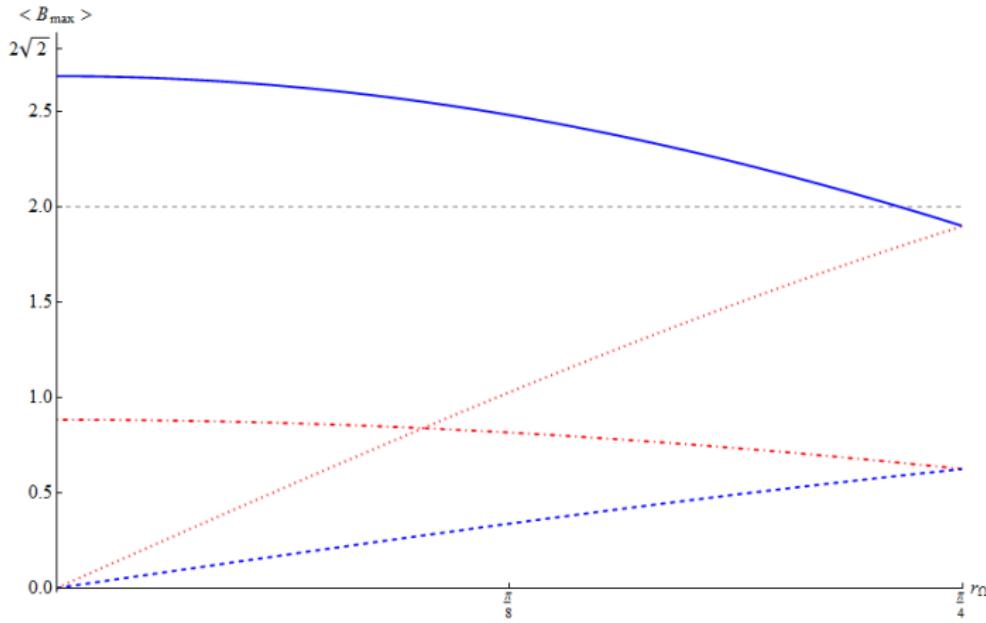
Nonlocality of Fermions

$$q_R = 1.00$$



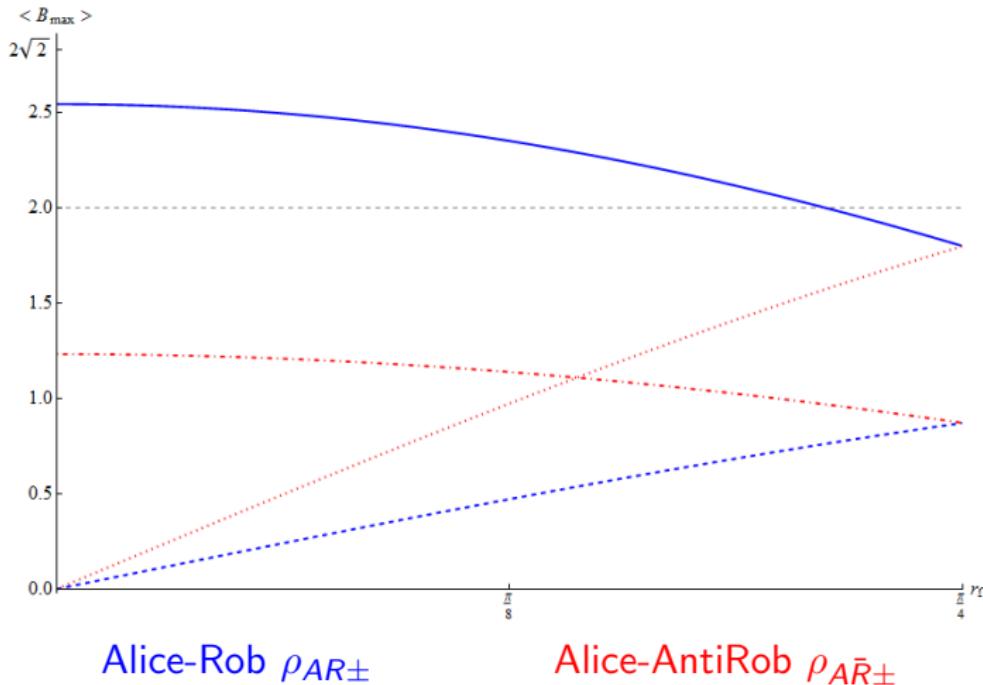
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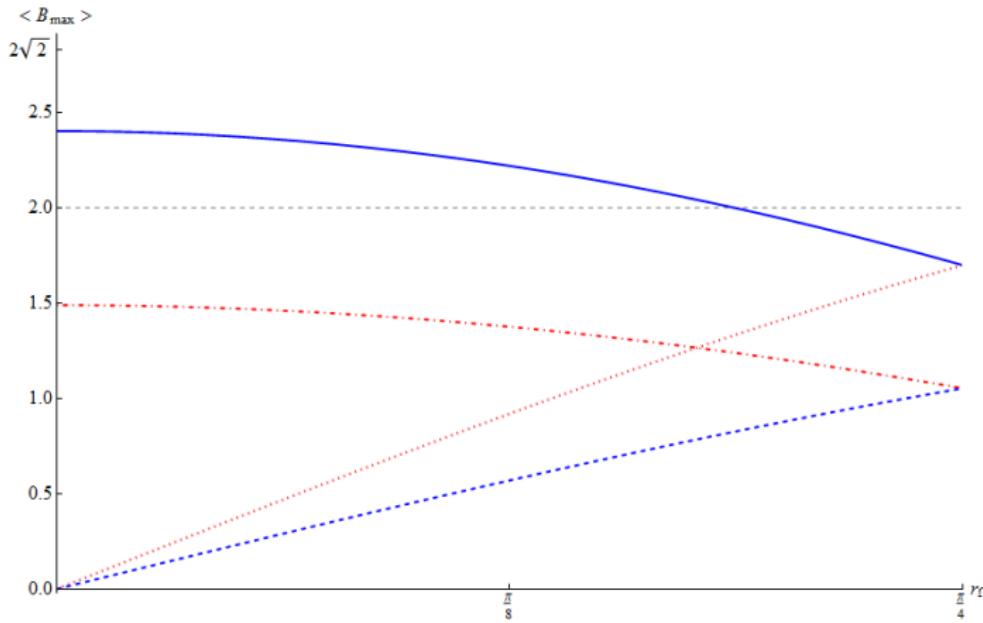
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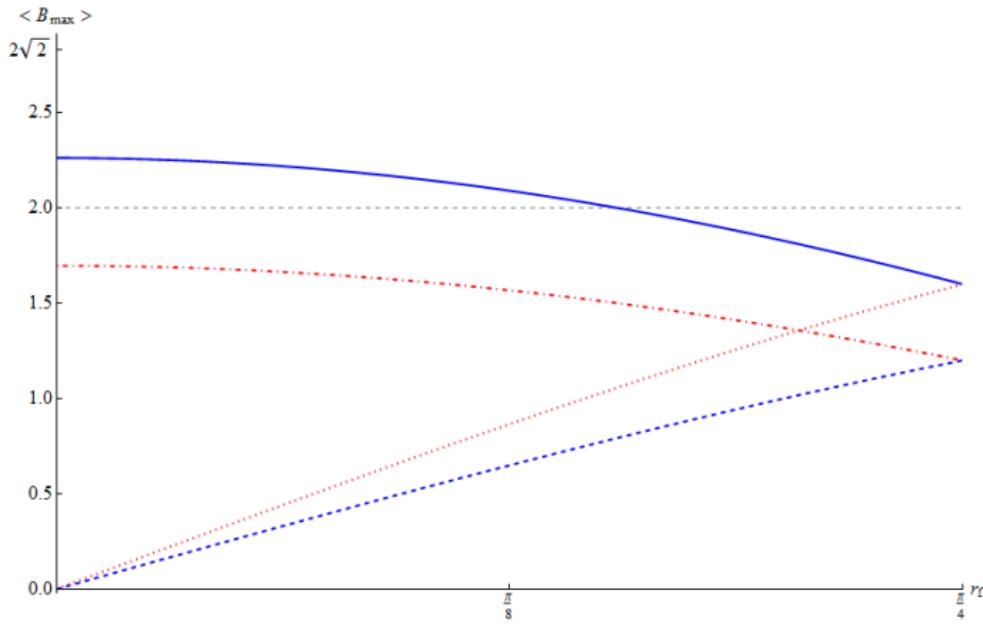
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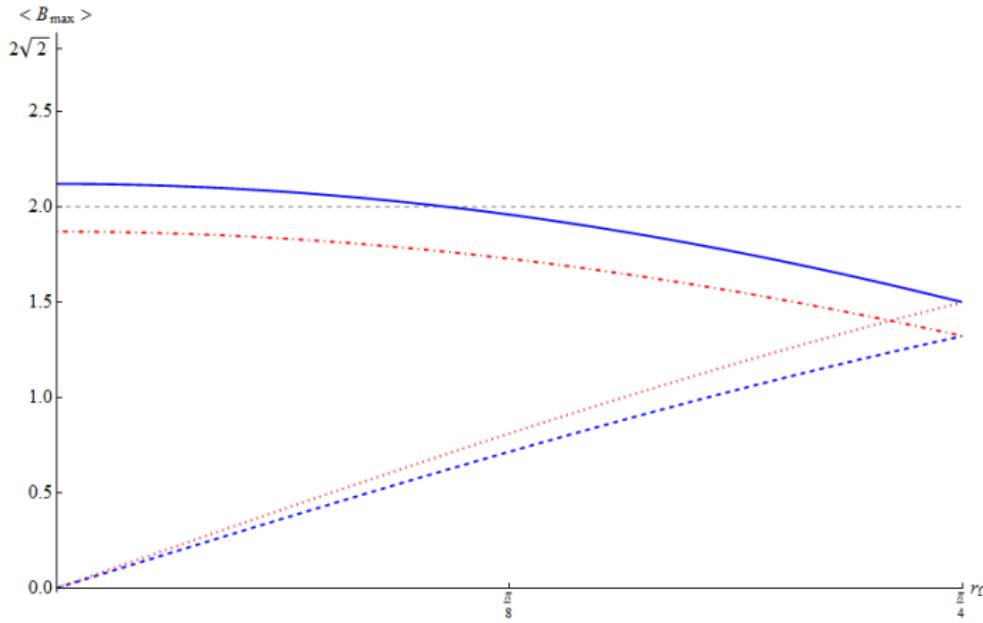
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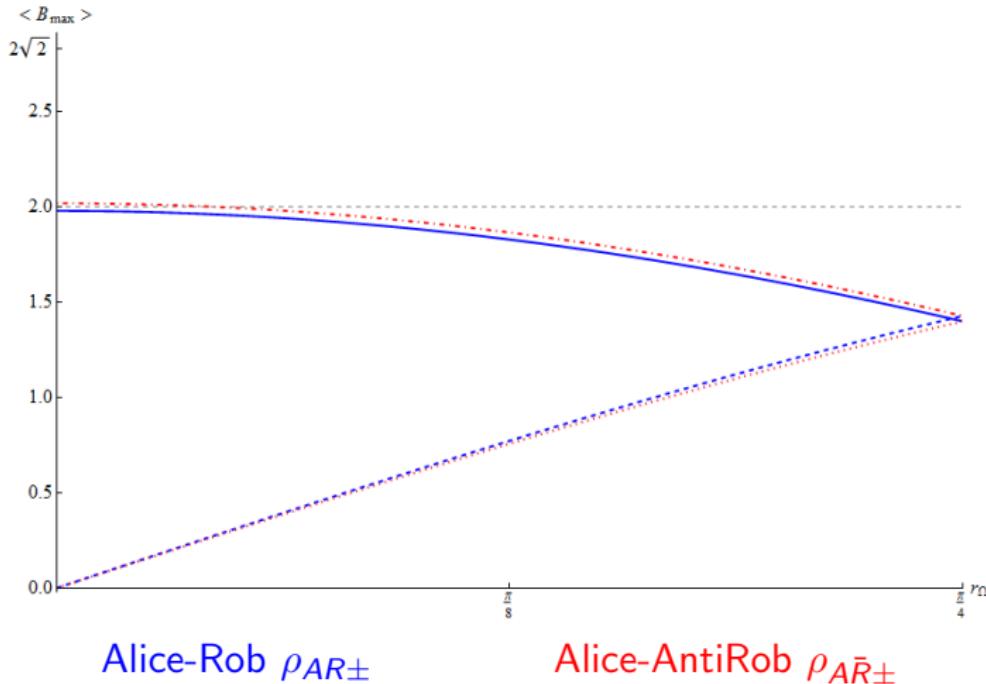
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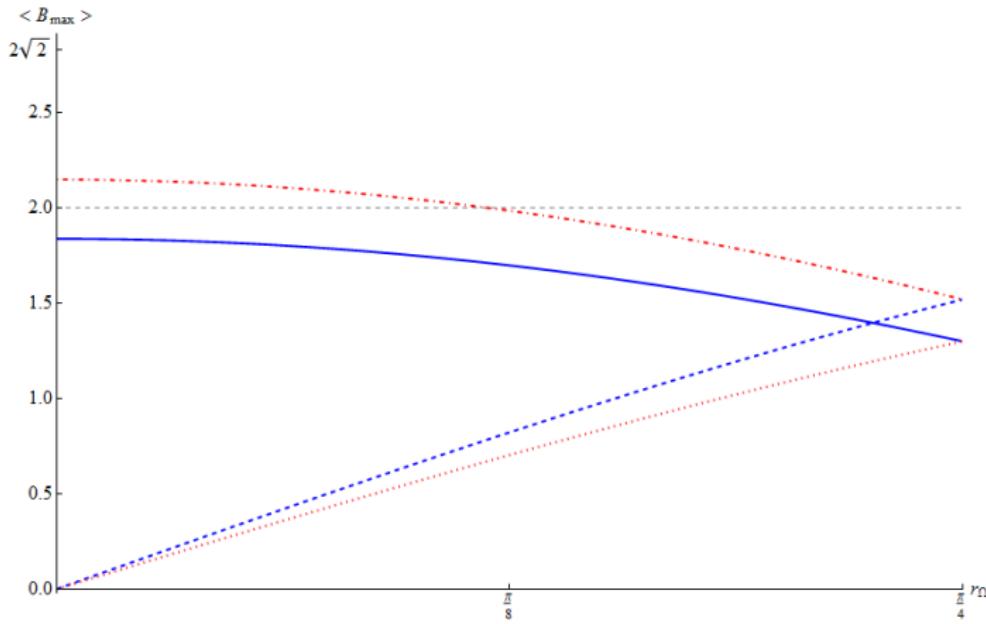
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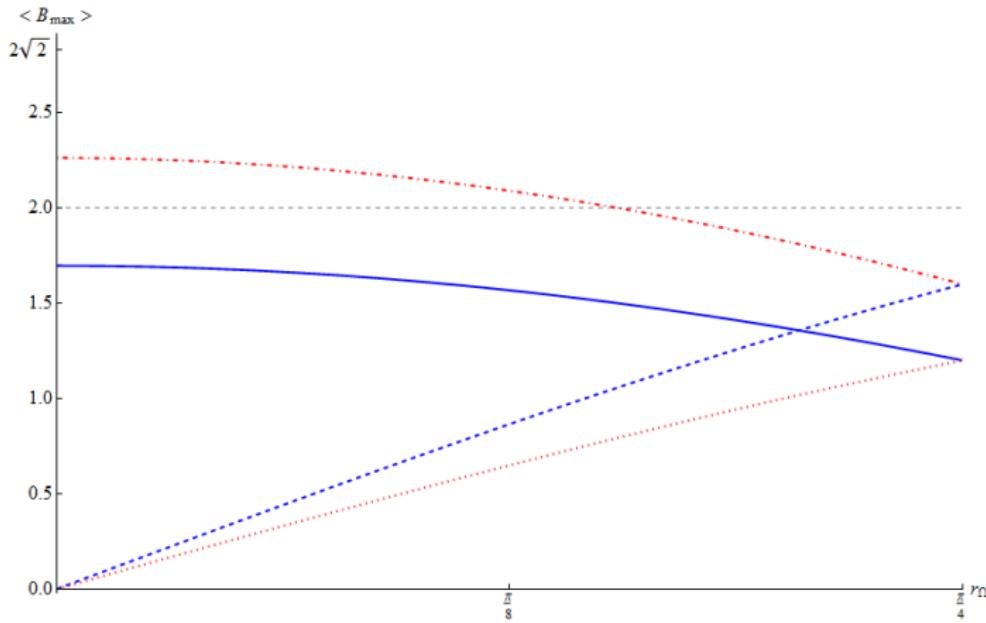
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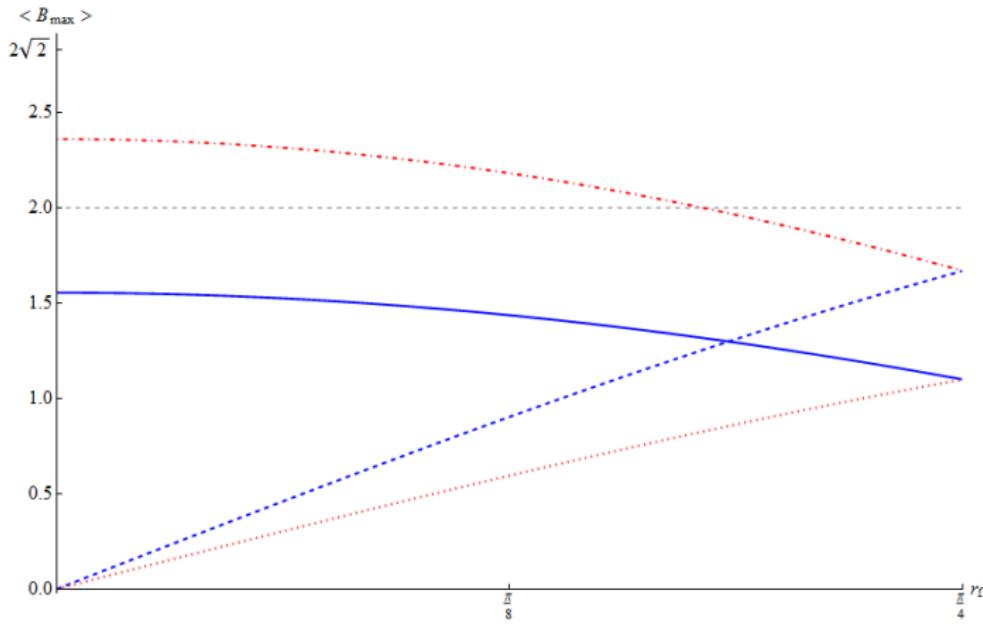
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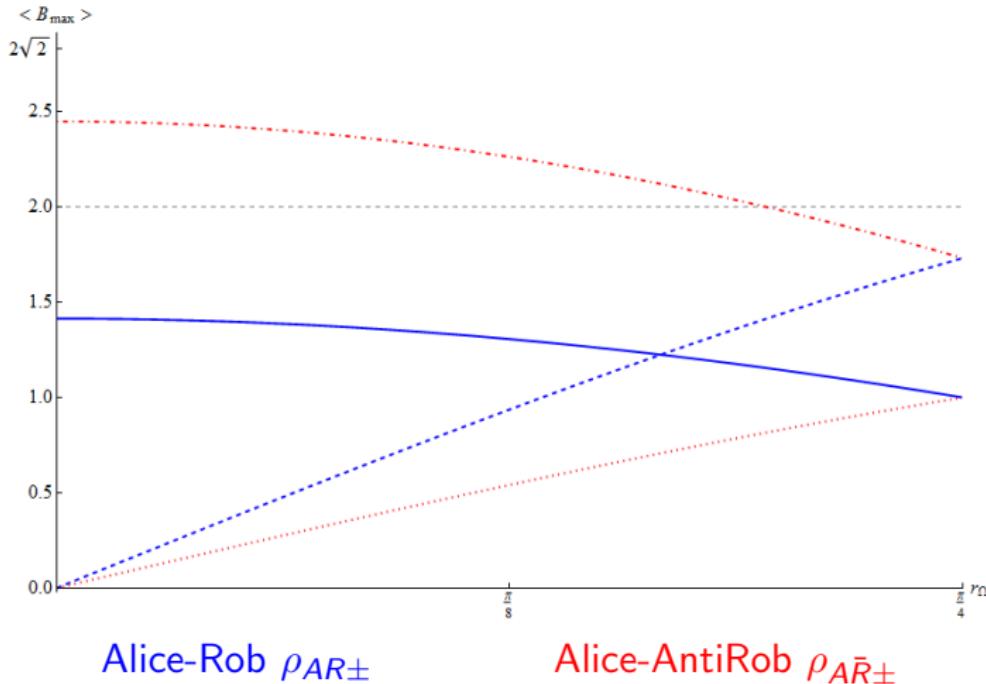
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Nonlocality of Fermions

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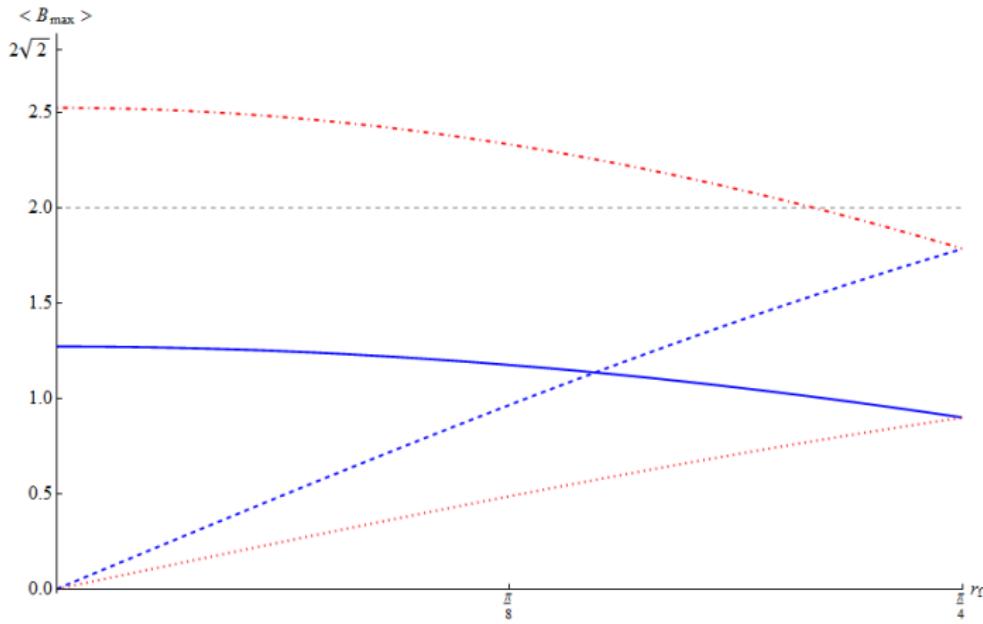
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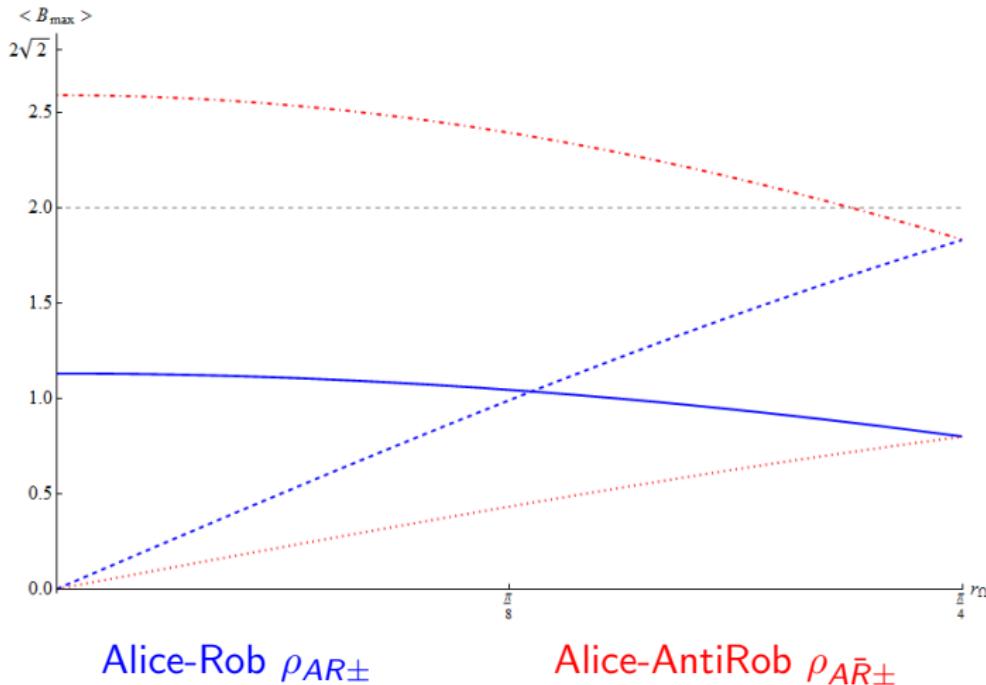
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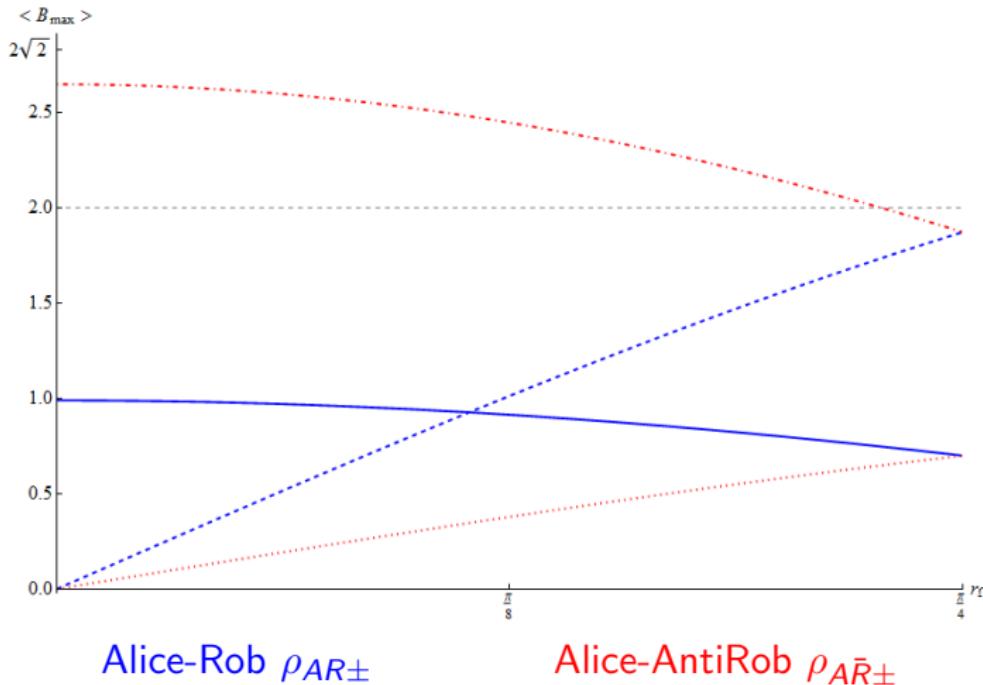
Alice-Rob $\rho_{A|R^\pm}$ Alice-AntiRob $\rho_{A|\bar{R}^\pm}$

Nonlocality of Fermions

 $q_R = 0.40$ 

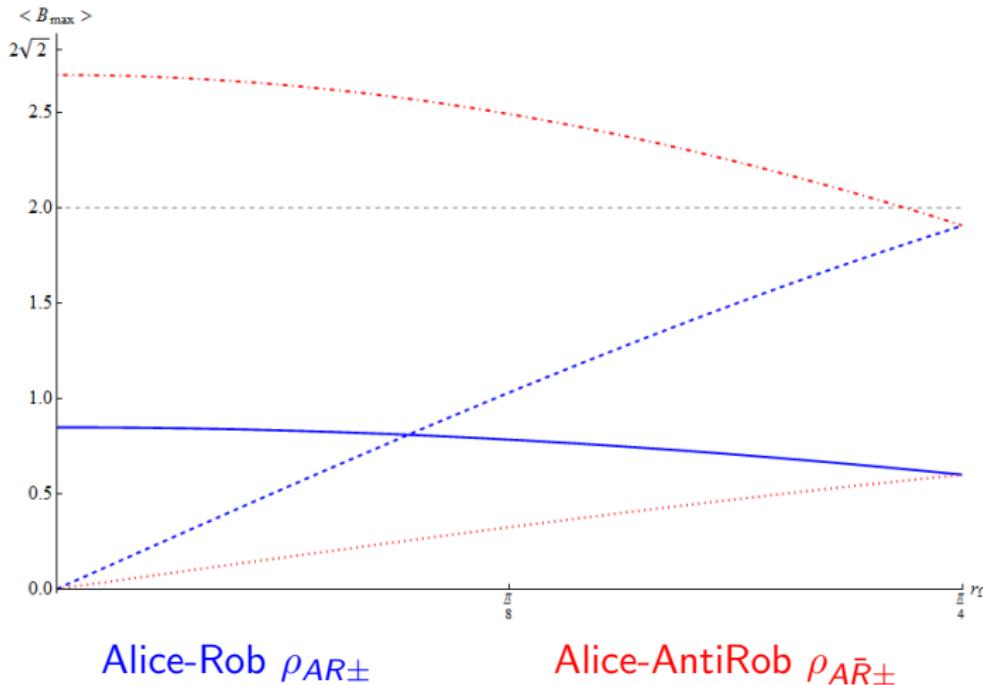
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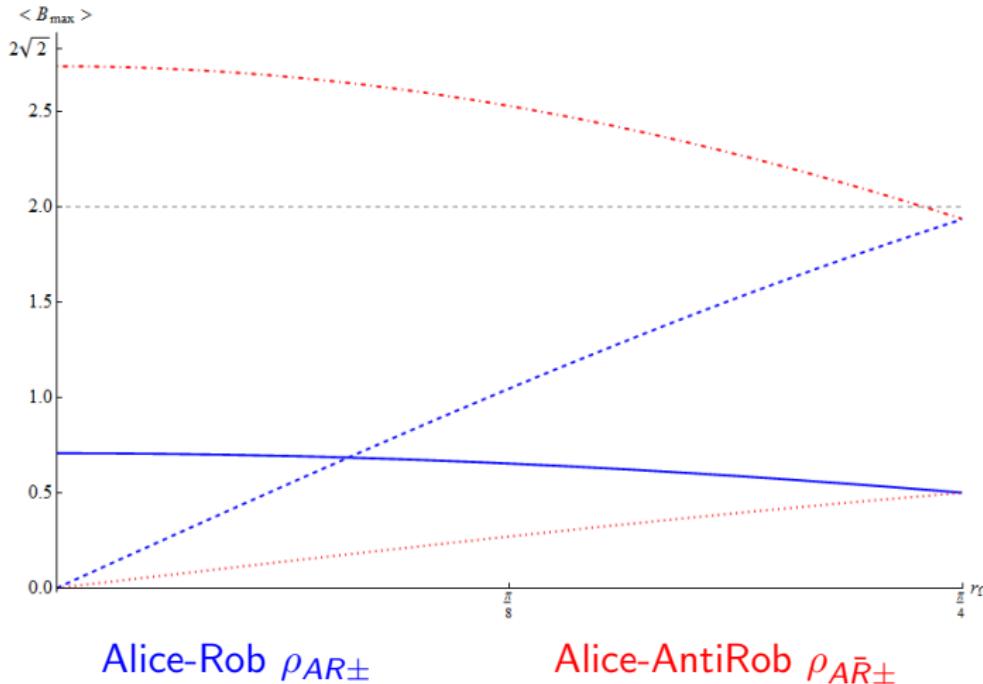
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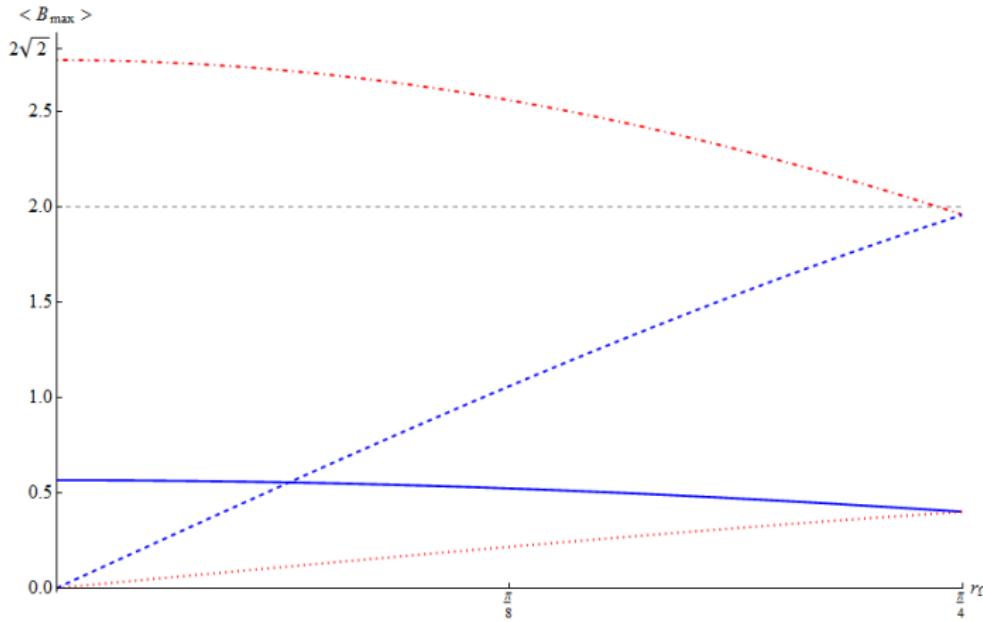


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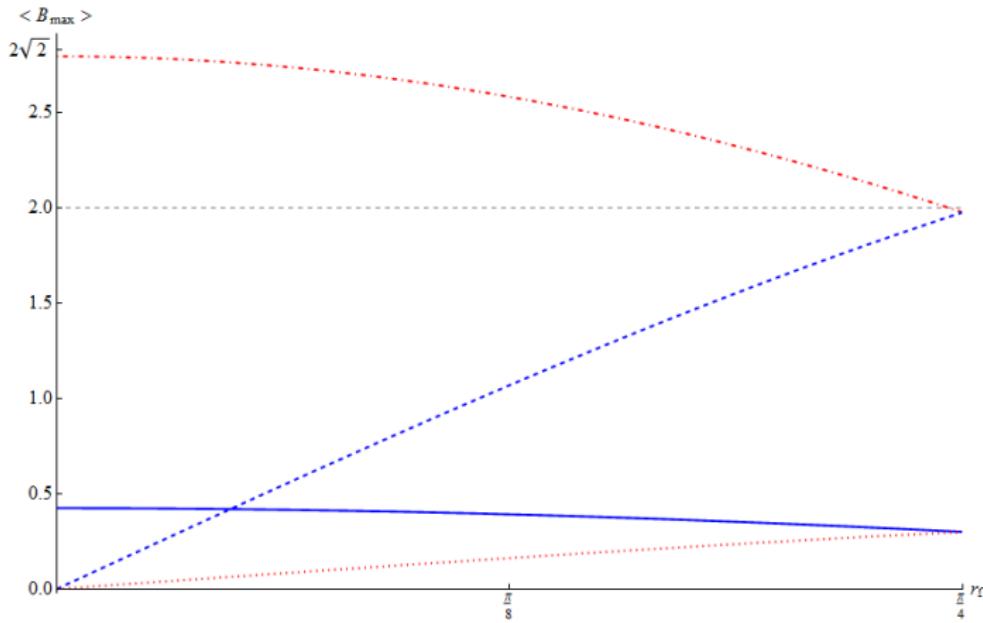
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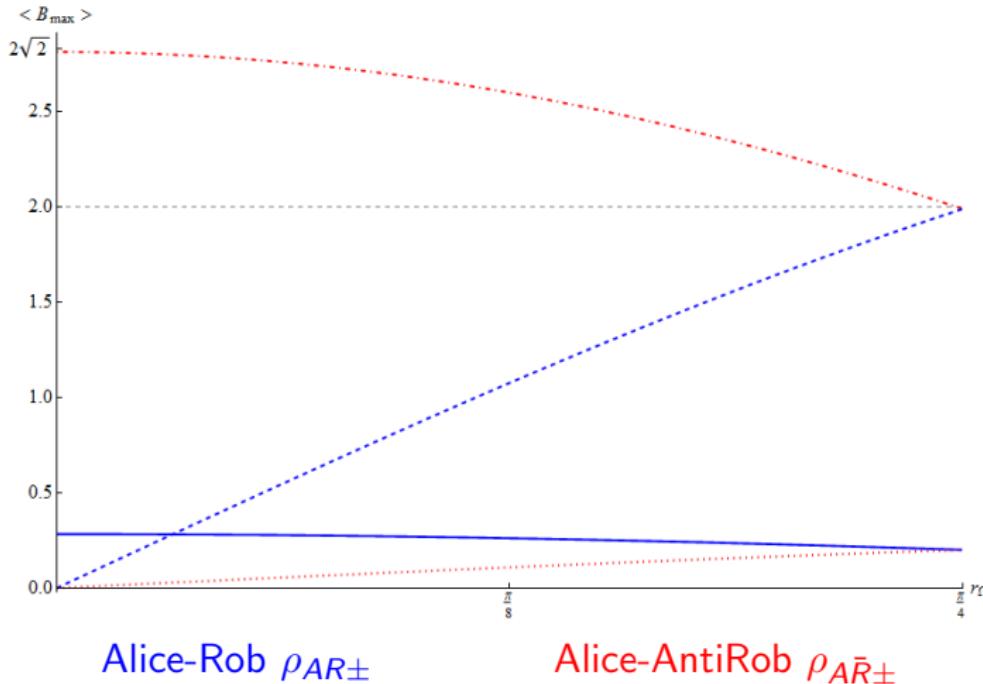
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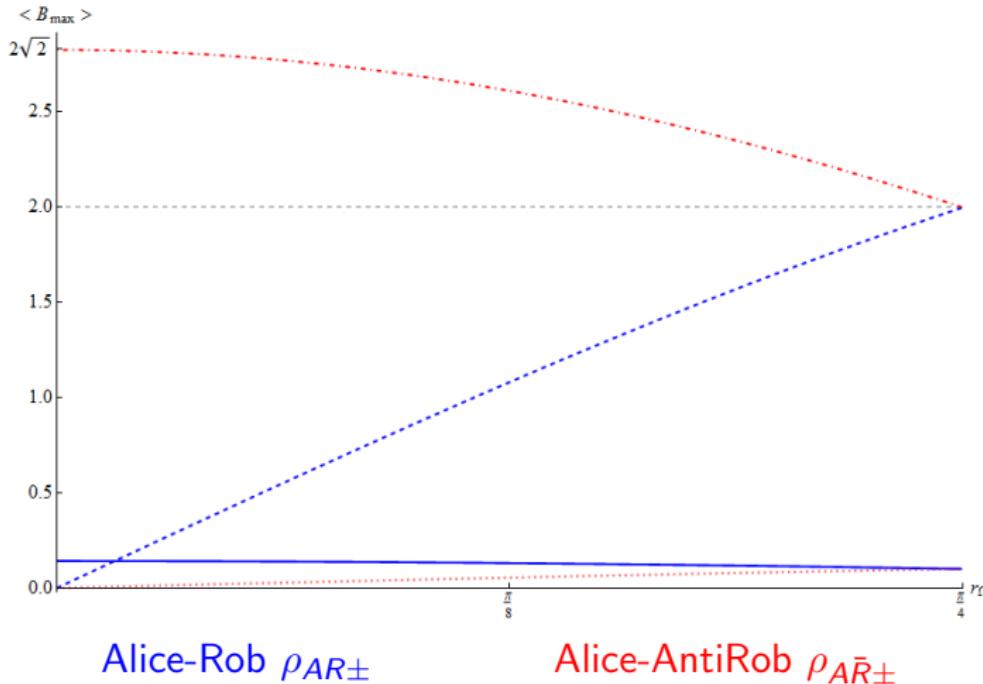
Nonlocality of Fermions

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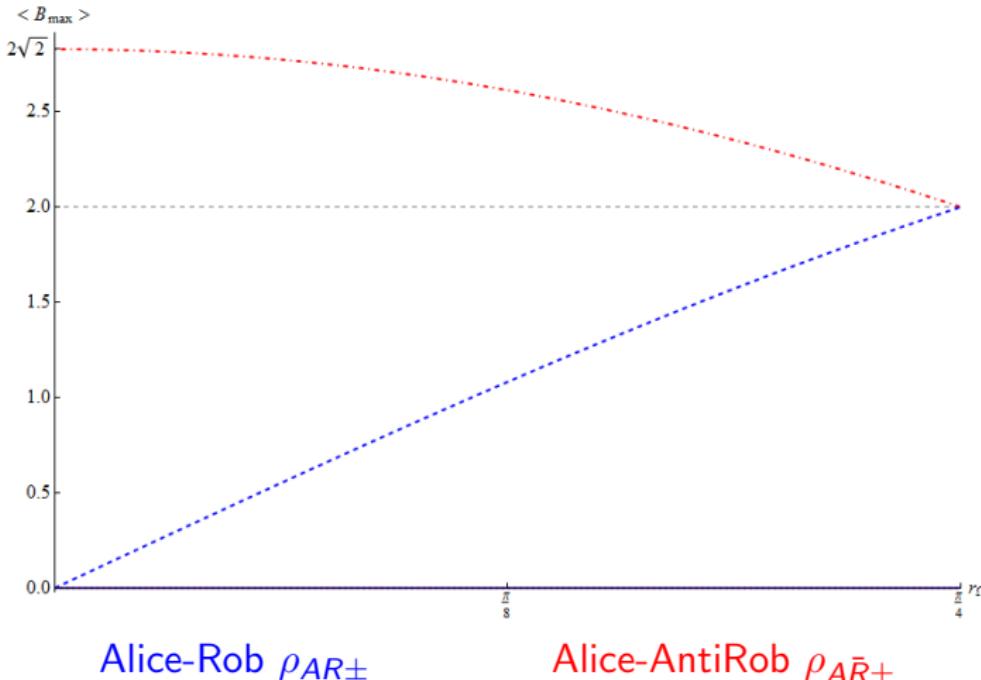
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Nonlocality of Fermions

 $q_R = 0.05$ 

Nonlocality of Fermions

 $q_R = 0.00$ 

Experimental issues

Preparation

- ▶ Preparation of **global mode** entanglement
- ▶ Preparation of **Unruh modes**
- ▶ **Eternal** acceleration
- ▶ **Large accelerations** required

Measurements

- ▶ Measurement of global modes
- ▶ **Change of basis** \Leftarrow swapping to different system

Conclusion

Summary

- ▶ Fermionic entanglement **non-vanishing** for infinite acceleration
- ▶ Fermionic non-locality **is vanishing** for infinite acceleration
- ▶ Only **one pair** of inertial vs. accelerated observers can violate CHSH inequality

Outlook & Open Questions

- ▶ How to prepare states & perform measurements?
- ▶ How can modes be localized?
⇒ “**Fermions in boxes**” (together with A. Lee, D. E. Bruschi and J. Louko, on the arXiv soon)



Thank you for your attention

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the University of Nottingham, CSIC, the QUITEMAD and the ChiQEN-Collaboration