



Quantum-enhanced deliberation of learning agents using trapped ions

Seventh International Workshop DICE 2014

Spacetime — Matter — Quantum Mechanics ... news on missing links

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Austrian Academy of Sciences

work in collaboration with *Vedran Dunjko* and *Hans J. Briegel*

Outline

Projective Simulation

- Projective simulation (PS) agents
- Standard PS vs. reflecting PS (RPS)
- The RPS deliberation process

Quantum-enhanced deliberation using trapped ions

- Flexible implementations — coherent controlization
- Trapped ion setups
- Coherent controlization using trapped ions

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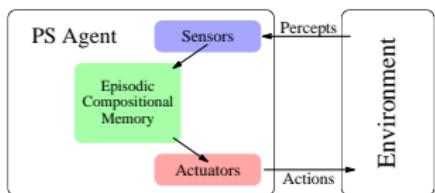
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Projective Simulation (PS) Agents¹

- embodied
- autonomous
- homogeneous
- active/passive

Episodic & compositional memory:
experience stored as clips

PS agents: deliberation process based
on a random walk in the clip space



Decision-making

- input (percept)
→ deliberation
→ output (action)

Represented by Markov chain with
transition matrix $P = [p_{ij}]$

Different types of deliberation processes
e.g., standard PS & reflecting PS

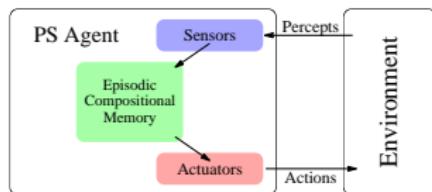
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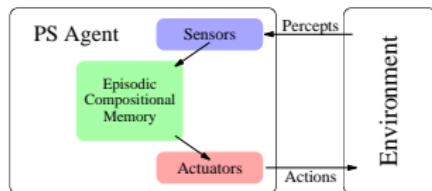
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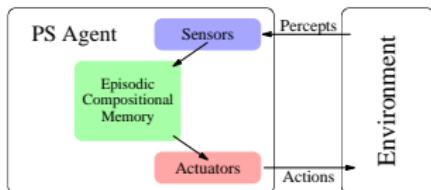
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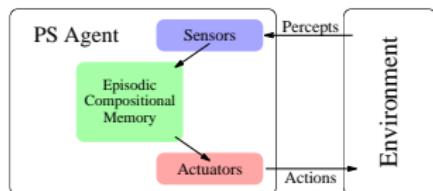
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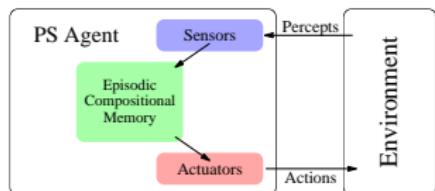
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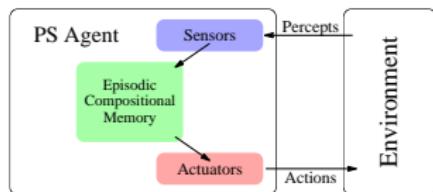
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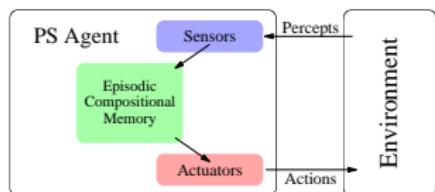
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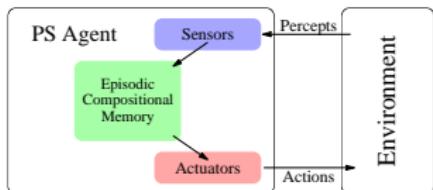
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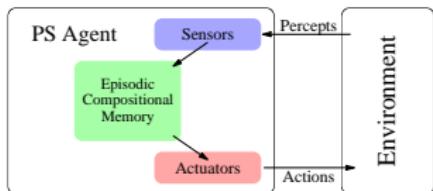
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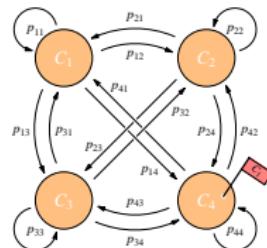
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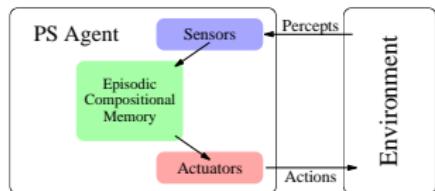
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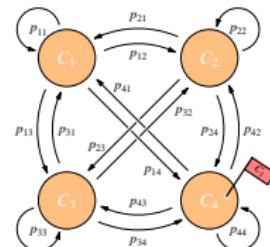
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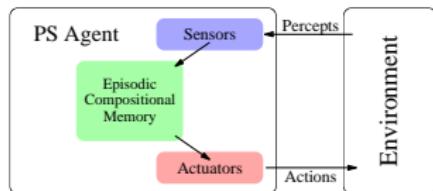
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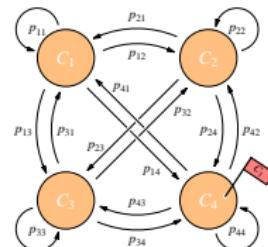
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Standard PS vs. Reflecting PS (RPS)

Standard PS

One clip network \Leftrightarrow Markov chain

Percept associated to clip i registered:
 random walk starting from clip i
 until action is obtained \rightarrow output

Quantization

clips: state vectors $|0\rangle, |1\rangle, |2\rangle, \dots$
 associate unitary U_i to each clip, s.t.,

$$U_i |0\rangle = \sum_j \sqrt{p_{ji}} |j\rangle$$

measure in clip-basis, result: $|k\rangle$
 if k is action \rightarrow output
 if not, apply U_k to $|0\rangle$, repeat

Reflecting PS²

Desire output according to distribution π
 specific to each percept
 Separate clip network for every percept
 Focus on one percept with transition
 matrix P , s.t., $P\pi = \pi$

Quantization

two registers I/II: $\{|i\rangle\}_{I/II}$
 use U_i to construct operators U_P, V_P :

$$U_P |i\rangle_I |0\rangle_{II} = |i\rangle_I U_i |0\rangle_{II}$$

$$V_P |0\rangle_I |i\rangle_{II} = U_i |0\rangle_I |i\rangle_{II}$$

Encode π : $|\pi'\rangle = \sum_i \sqrt{\pi_i} |i\rangle_I U_i |0\rangle_{II}$

Szegedy-walk operator³ $W(P)$:
 $W(P) |\pi'\rangle = |\pi'\rangle$

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Standard PS vs. Reflecting PS (RPS)

Standard PS

One clip network \Leftrightarrow Markov chain

Percept associated to clip i registered:
random walk starting from clip i
 until **action** is obtained \rightarrow output

Quantization

clips: state vectors $|0\rangle, |1\rangle, |2\rangle, \dots$
 associate unitary U_i to each clip, s.t.,

$$U_i |0\rangle = \sum_j \sqrt{p_{ji}} |j\rangle$$

measure in clip-basis, result: $|k\rangle$
 if k is **action** \rightarrow output
 if not, apply U_k to $|0\rangle$, repeat

Reflecting PS²

Desire **output** according to **distribution π**
 specific to each percept

Separate clip network for every percept

Focus on one percept with transition
 matrix P , s.t., $P\pi = \pi$

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two registers I/II: $\{|i\rangle\}_{I/II}$

use U_i to construct operators U_P, V_P :

$$U_P |i\rangle_I |0\rangle_{II} = |i\rangle_I U_i |0\rangle_{II}$$

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The RPS Decision-Making Process

RPS agent: output actions according to tail of stationary distribution π

Achieved by Grover-like procedure!²

Recall Grover's algorithm⁴

Search a set \mathcal{S} for marked items
from a subset $\mathcal{M} \subset \mathcal{S}$

- (0) Prepare $|\psi\rangle$: uniform superposition of basis states representing all items
- (1) Reflection over marked items $|\phi\rangle$
- (2) Reflection over $|\psi\rangle$

Repeat steps (1) and (2)

Note: every item found equally likely

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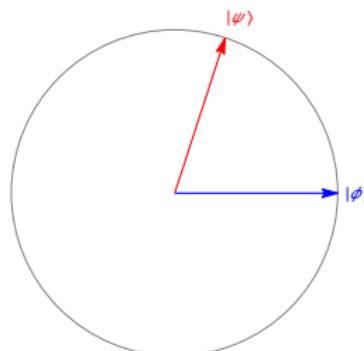
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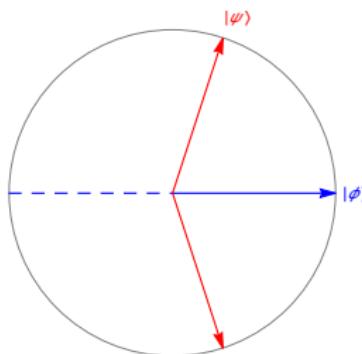
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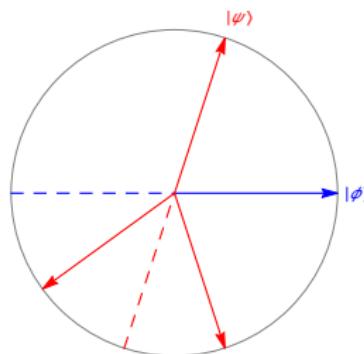
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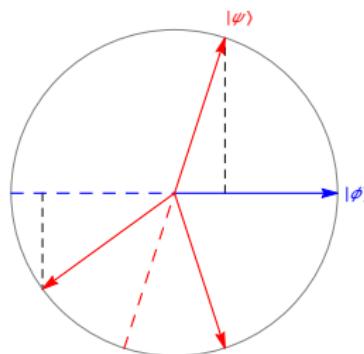
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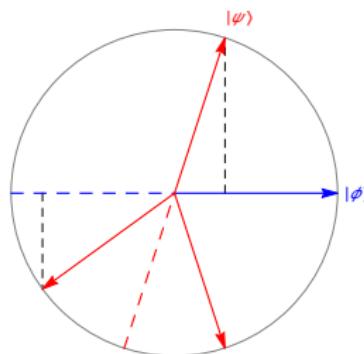
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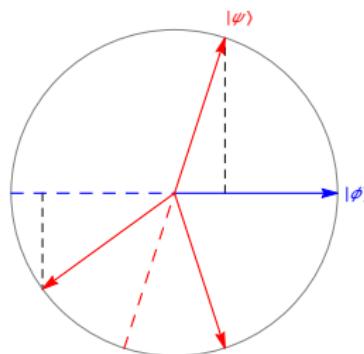
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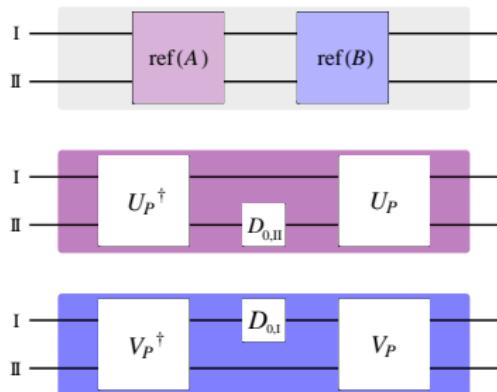
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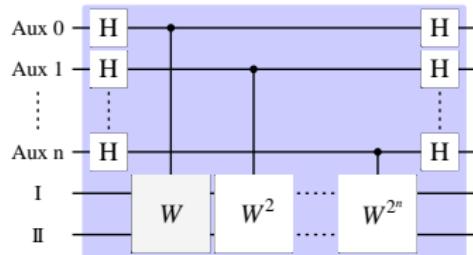
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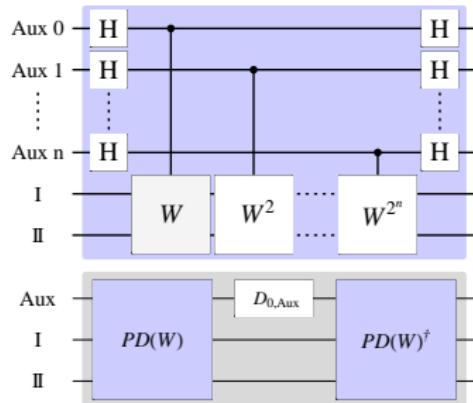
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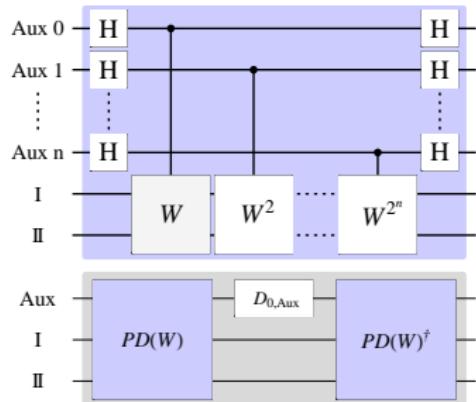
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- Repeat steps (1) and (2)



Sample, and repeat if no action is found.

Quantum RPS quadratically faster than classical RPS! Speed-up!

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The RPS Decision-Making Process

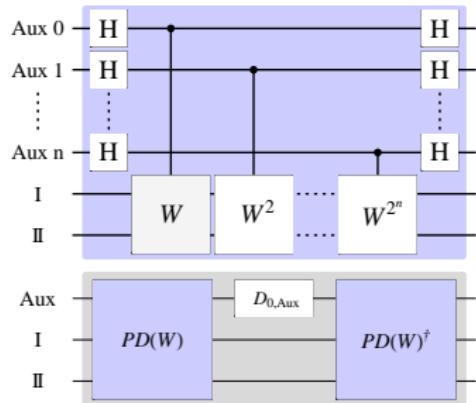
RPS agent: output actions according to tail of stationary distribution π

Approximate reflection:

$$U_i \rightarrow U_P, V_P \rightarrow W(P) \rightarrow PD(W) \rightarrow ARO$$

RPS: Grover-like procedure

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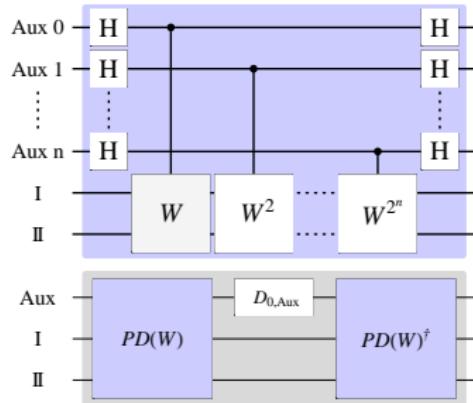
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Flexible Implementation — Coherent Controlization

Key element: **flexible** (updateable) implementation without prior knowledge of π

First step: construction of unitaries U_i : first column: real, positive entries $\sqrt{p_{ji}}$

$$\text{For two clips: } U(\theta) = \exp\left(-i \frac{\theta}{2} Y\right) = \begin{pmatrix} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} \\ \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix}$$

Extend by nested scheme of adding control⁶, e.g., for (up to) 8 clips:

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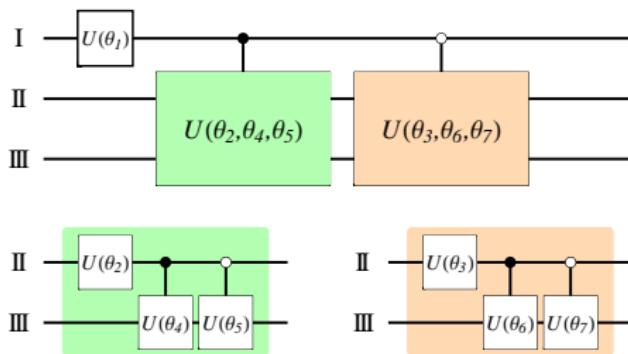
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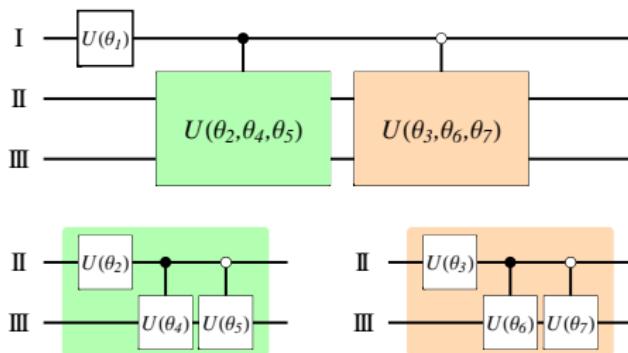
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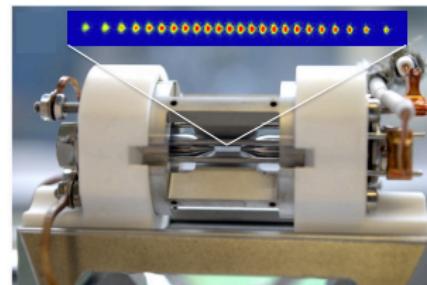
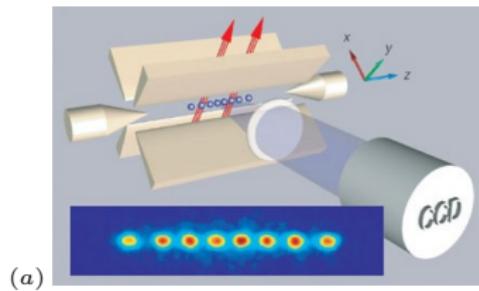


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Ion Trap Setups

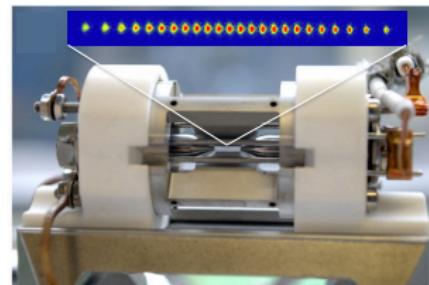
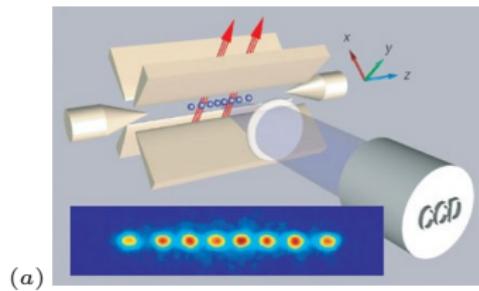


String of $^{40}\text{Ca}^+$ ions in linear quadrupole trap — Paul trap

- Electric fields applied by blades (rapidly oscillating) & tips (static)
⇒ nearly harmonic 3-d trapping potential
- Distance between ions $\approx \mu\text{m}$ ⇒ ions individually addressable
- Coulomb coupling ⇒ collective vibrational modes

- (a) Figure from: ⁸ Blatt & Wineland, *Entangled states of trapped atomic ions*, Nature **453**, 1008–1015 (2008).
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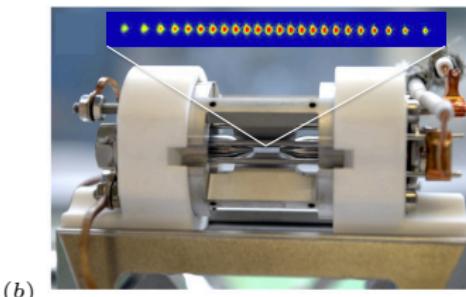
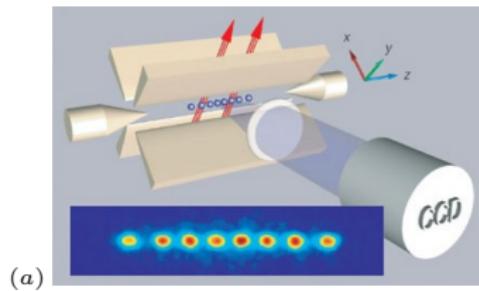


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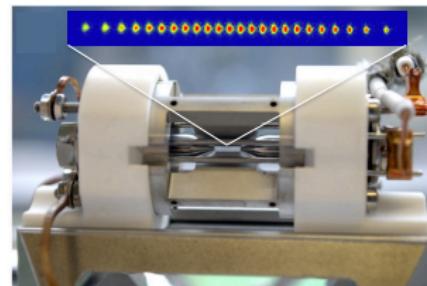
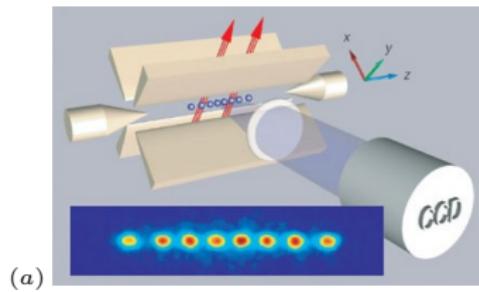


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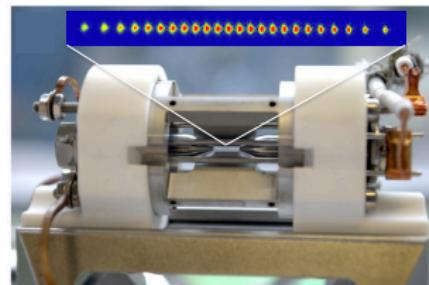
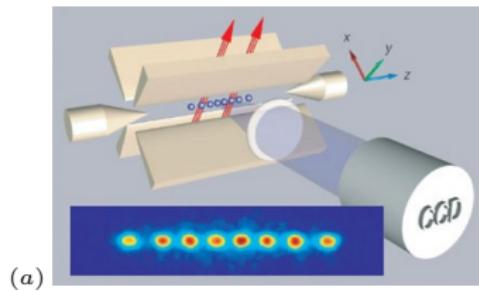


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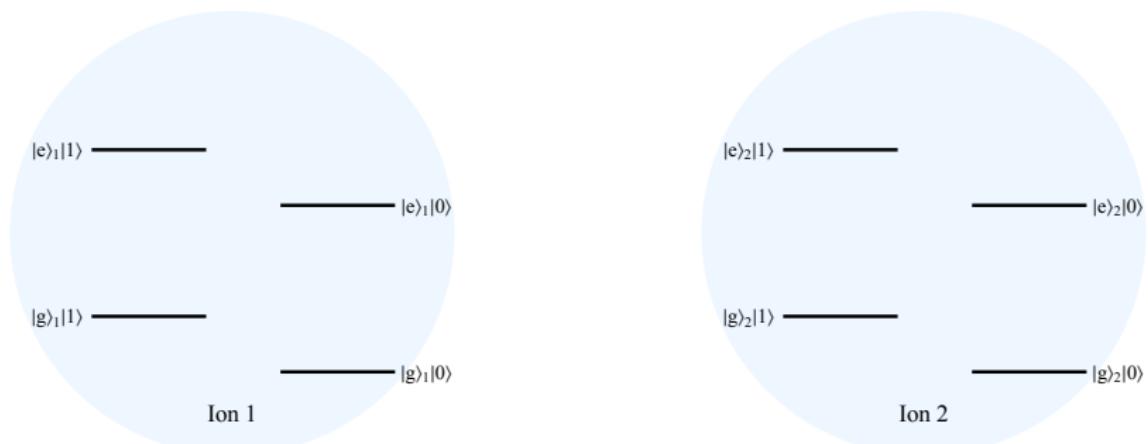
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Scheme to Add Control to Unknown Unitaries⁷

Initial state: $(\alpha |g\rangle_1 + \beta |e\rangle_1) |\psi\rangle_2 |0\rangle$

(i) Cirac-Zoller method⁹: laser pulse on ion 1: $|g\rangle_1|0\rangle \rightarrow |e\rangle_1|1\rangle$



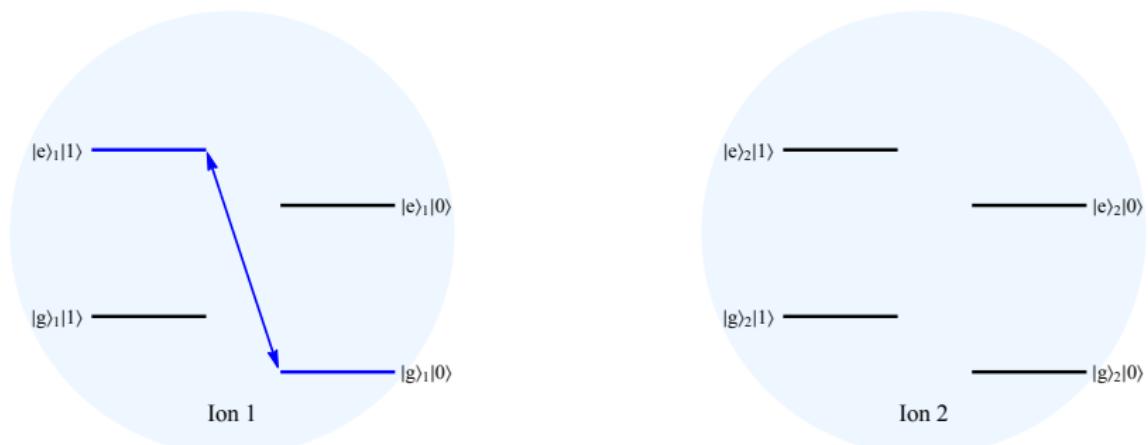
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$$(\alpha |g\rangle_1 + \beta |e\rangle_1) |\psi\rangle_2 |0\rangle \rightarrow |e\rangle_1 |\psi\rangle_2 (\alpha |1\rangle + \beta |0\rangle)$$

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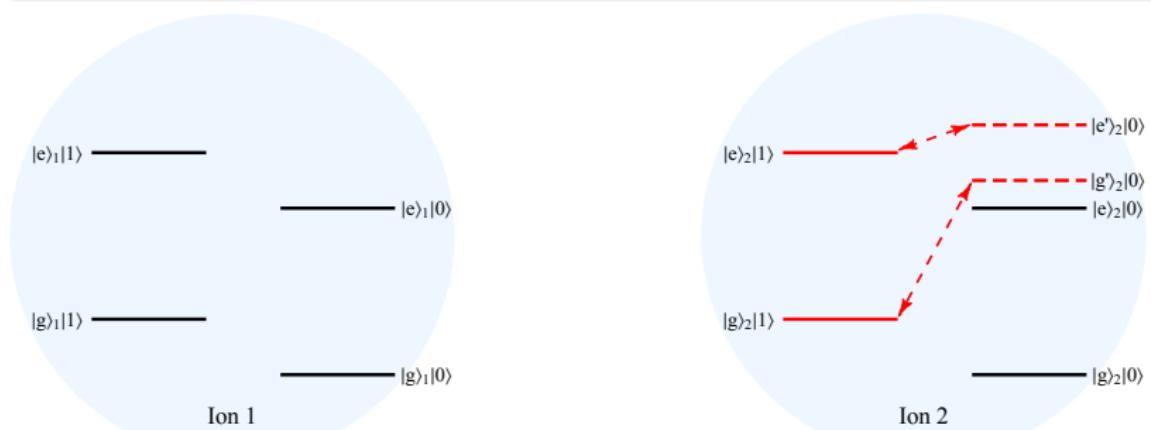
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$$|e\rangle_1 |\psi\rangle_2 (\alpha |1\rangle + \beta |0\rangle) \rightarrow |e\rangle_1 (\alpha |\psi'\rangle_2 + \beta |\psi\rangle_2) |0\rangle$$

(ii) **Hiding pulses:** laser pulses on ion 2: $|g\rangle_2 |1\rangle \rightarrow |g'\rangle_2 |0\rangle$



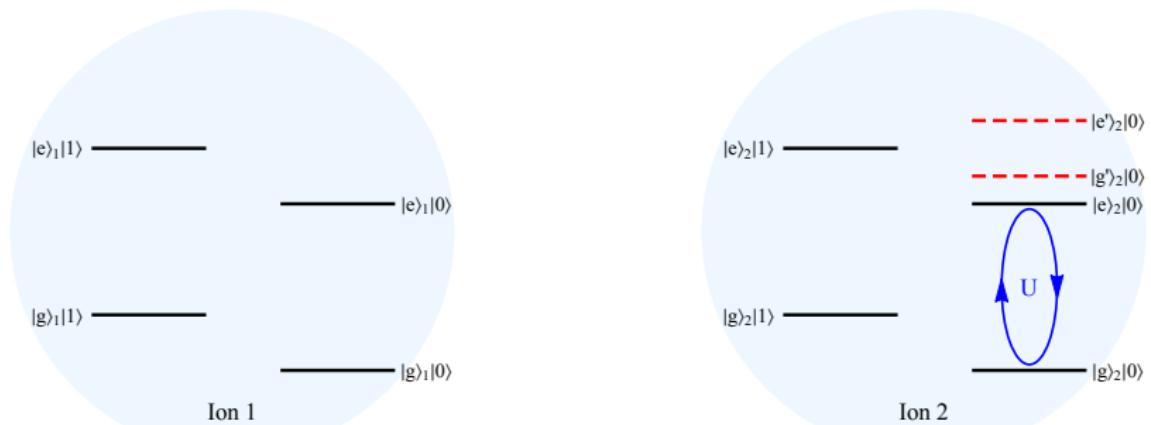
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$$|e\rangle_1 (\alpha |\psi'\rangle_2 + \beta |\psi\rangle_2) |0\rangle \rightarrow |e\rangle_1 (\alpha |\psi'\rangle_2 + \beta U |\psi\rangle_2) |0\rangle$$

(iii) **Unknown unitary U :** laser pulse on ion 2: $|\psi\rangle_2 \rightarrow U |\psi\rangle_2$



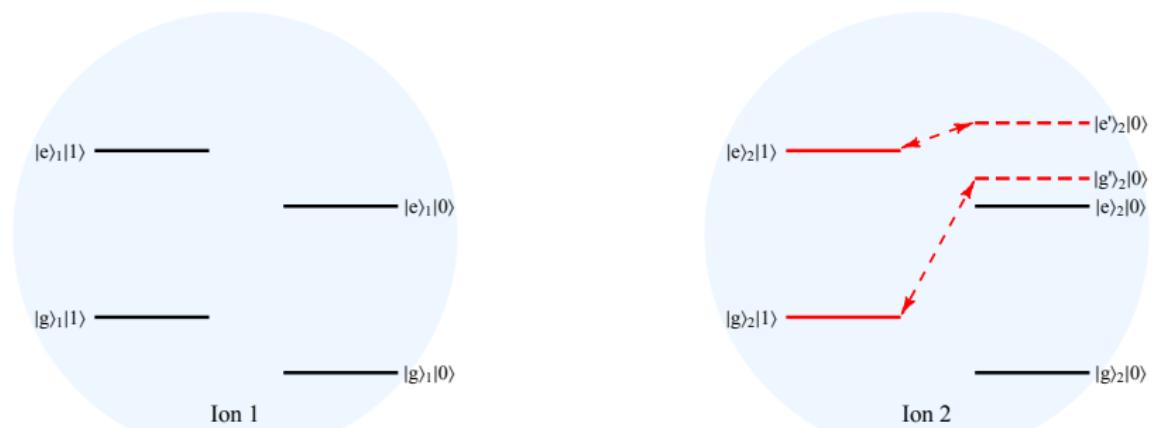
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(iv) **Undo hiding pulses:** laser pulses on ion 2: $|g'\rangle_2 |0\rangle \rightarrow |g\rangle_2 |1\rangle$



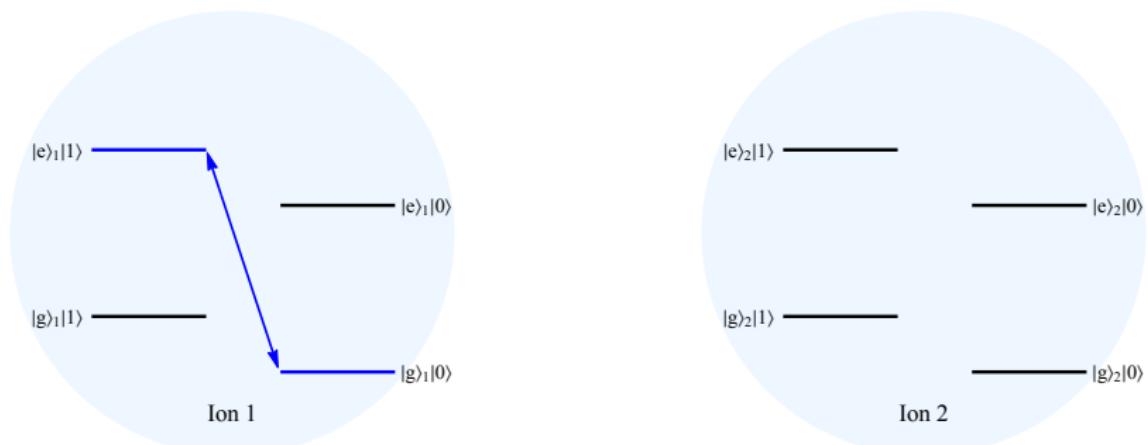
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Scheme to Add Control to Unknown Unitaries⁷

$$|e\rangle_1 (\alpha|\psi\rangle_2 |1\rangle + \beta \textcolor{blue}{U} |\psi\rangle_2 |0\rangle) \rightarrow (\alpha|g\rangle_1 |\psi\rangle_2 + \beta |e\rangle_1 \textcolor{blue}{U} |\psi\rangle_2) |0\rangle$$

(v) **Switch back control:** Cirac-Zoller pulse⁹ on ion 1: $|e\rangle_1 |1\rangle \rightarrow |g\rangle_1 |0\rangle$



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- Feasible implementation: detailed simulations including errors for simple scenario⁶

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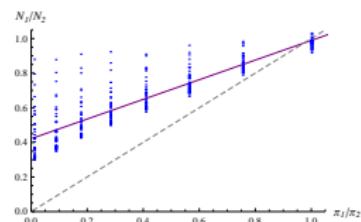
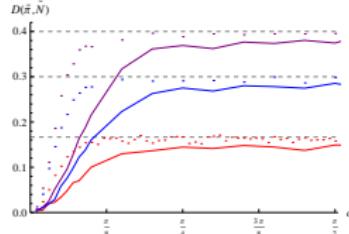
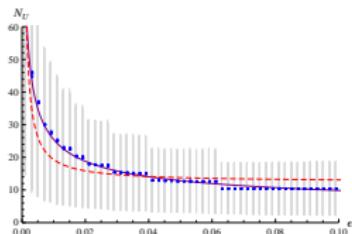
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